SUPER FELICITOUS DIFFERENCE LABELING GRAPH

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Abstract
A graph with p vertices and q edges is called super felicitous difference labeling graph if f: V (G) → $\{1,2,\ldots\ldots p+q\}$ is an injective map so that the induced edge labeling is defined by $f^*(e=uv)=\big(f(u)-f(v)\big)$ (mod $p+q$)and$f\big(v(G)\big)\cup f^*(e): e\in E(G)=\{1,2,3\ldots.p+q\}$. In this paper, we prove that Path, Cycle, Complete Bipartite Graph, Fan, Comb, $B_{n,n}(n\geq 2)$, Star graph are Super Felicitous Difference Labeling Graph. As a consequence, some families of graphs are shown to be non – Super felicitous difference labeling.
Keywords: Super Felicitous Difference Labeling (SFDL)

I.INTRODUCTION

The graphs we consider are simple. For notation and terminology, we refer to [3]. Several graph labelings have been found in Gallian survey [4]. Lee and schmeichel and shee [6] introduced the concept of felicitous graph as a generalization of harmonious graph. V. Lakshmi Alias Gomathi, A. Nagarajan, A. Nellai Murugan introduced the concept of felicitous labeling of a graph. Dr. A. Punitha Tharani and E.S.R. Francis Vijaya Rani introduced the concept of felicitous difference labeling graph [1].

R. Ponraj and D. Ramya introduced the concept of Super mean graph [5]. Super Felicitous labeling graph was introduced by V. Lakshmi Alias Gomathi, A. Nagarajan, A. Nellai Murugan. It motivates us to define the concept of Super Felicitous Difference Labeling graph.

II. Preliminaries

Definition 2.1: A **Path P_n** is a walk in which all the vertices are distinct.
Definition 2.2: A **Cycle C_n** is a Closed Path.
Definition 2.3: A **Complete Bipartite graph $k_{m,n}$**is a bipartite graph with bipartition (V_1,V_2)such that every vertex of V_1 is joined to all the vertices of V_2, Where $|V_1|$ = m and
$|V_2|$ = n.
Definition 2.4: The **fan f_n** (n ≥ 2) is obtained by joining all vertices of P_n (Path of n vertices) to a further vertex called the center and contains n + 1 vertex and 2n-1 edges. i.e. $f_n = p_n + k_1$.
Definition 2.5: The graph obtained by joining a single pendant edge to each vertex of a path is called a **Comb.**
Definition 2.6: The **Bistar $B_{m,n}$**is the graph obtained from k_2 by identifying the center vertices of $k_{1,m}$ and $k_{1,n}$ at the end vertices of k_2 respectively. $B_{n,n}$ is often denoted by B(n).
Definition 2.7: The Complete Bipartite Graph $k_{1,n}$ is called a **Star Graph** and it is denoted by S_m.

III. Main Results

Definition 3.1: A graph with p vertices and q edges is called super felicitous difference labeling graph if f: V (G) → $\{1,2,\ldots\ldots p+q\}$ is an injective map so that the induced edge labeling is defined by $f^*(e=uv)=\big(f(u)-f(v)\big)$ (mod $p+q$) and $f\big(v(G)\big)\cup f^*(e): e\in E(G)=\{1,2,3\ldots.p+q\}$. A graph that admits **Super Felicitous Difference Labeling** (SFDL) is called **Super Felicitous Difference Labeling Graph.**
Theorem 3.2: Any Path $P_n$is a SFDL graph for any n≥ 2.
Proof: Let V (P_n) = $\left\{{}^{u_i}/_{1\leq i\leq n}\right\}$ and E (P_n) =$\left\{{}^{u_iu_{i+1}}/_{1\leq i\leq n-1}\right\}$. Then |V ($P_n$)| = n and |E ($P_n$)| = n – 1. Define f:V ($P_n$) → $\{1,2,\ldots\ldots 2n-3,2n-1\}$ as follows:
Case: (i): When n is odd.
$f(u_1)=2n-1$,

$f(u_{2k+1}) = 2n - i, \quad 1 \le k \le \frac{n-2}{2}\ , 1 \le i \le n-1$

$f(u_{2k}) = 2k - 1, \quad 1 \le k \le \frac{n}{2}$

Case: (ii): When n is even.

$f(u_1) = 2n - 1,$

$f(u_{2k+1}) = 2n - i, \quad 1 \le k \le \frac{n-1}{2}\ , 1 \le i \le n$

$f(u_{2k}) = 2k - 1, \quad 1 \le k \le \frac{n-1}{2}$

let f^* be the induced edge labeling of f. Then

$f^*(u_i, u_{i+1}) = 2n - 2i, \ \ 1 \le i \le n-1$

Cleary the labels V(G) ∪ E(G) are distinct. Hence from the above labeling pattern, the graph P_n admits SFDL graph.

Example 3.3: The SFD labeling graph of $\boldsymbol{P_8}$ is given in fig. 1

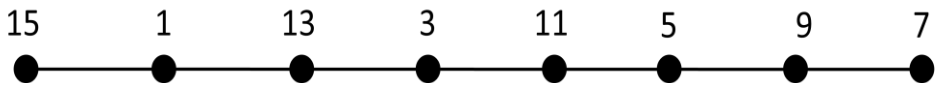

Fig. 1

Theorem 3.4: Any Cycle $\boldsymbol{C_n}$is a SFDL graph for any n ≥ 3.

Proof: Case – (i): n = 2k + 1.

Let V (C_{2k+1}) = $\left\{{u, v, u_i}/_{1 \le i \le 2k+1}\right\}$ and E (C_{2k+1}) = $\left\{{u_i u_{i+1}}/_{1 \le i \le 2k-2}\right\} \cup \{(uv)\} \cup \{(uu_1)\} \cup \{(vu_{n-2})\}$.

Define f:V $(C_{2k+1}) \to \{1,2, \dots .2n-2, 2n\}$ as follows:

$f(u) = 2n,$

$f(v) = 2n - 2,$

$f(u_{2i+1}) = (2i + 1), \quad 0 \le i \le \frac{n-1}{2}$

$f(u_{2i}) = 2n - (2j + 1), \quad 0 \le j \le \frac{n-3}{2}$

let f^* be the induced edge labeling of f. Then

$f^*(uv) = 2$

$f^*(u, u_1) = 2n - 1$

$f^*(u_i, u_{i+1}) = 2n - 4 - 2j, \ \ 0 \le j \le n-2$

$f^*(v, u_{n-2}) = 2j + 1, \ \ 1 \le j \le n$

Cleary the labels V(G) ∪ E(G) are distinct. Hence from the above labeling pattern, the graph C_{2k+1} admits SFDL graph.

Example 3.5: The SFD labeling graph of $\boldsymbol{c_5}$ is given in fig. 1

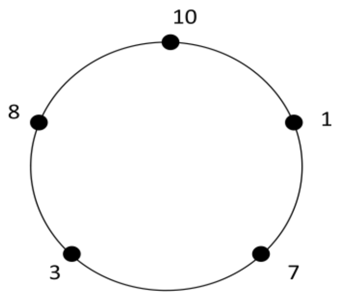

Fig. 2

Case – (ii): n = 2k

Let V (C_{2k}) = $\left\{{u, v, u_i}/_{1 \le i \le 2k-2}\right\}$ and E (C_{2k}) = $\left\{{u_i u_{i+1}}/_{1 \le i \le 2k-3}\right\} \cup \{(uv)\} \cup \{(uu_1)\} \cup \{(vu_{n-2})\}$. Define f:V $(C_{2k+1}) \to \{1,2, \dots .2n-2, 2n\}$ as follows:

$f(u) = 2n,$

$f(v) = 2,$

$f(u_{2i+1}) = 2i + 1, \quad 0 \leq i \leq \frac{n-4}{2}$

$f(u_{2i}) = 2n - (2j + 1), \quad 1 \leq j \leq \frac{n-2}{2}$

Let f^* be the induced edge labeling of f. Then

$f^*(uv) = 2n - 2$

$f^*(u, u_1) = 2n - 1$

$f^*(u_i, u_{i+1}) = 2n - 4 - 2j, \; 0 \leq j \leq n - 2$

$f^*(v, u_{n-2}) = 2j + 1, \; 1 \leq j \leq n$

Cleary the labels V(G) ∪ E(G) are distinct. Hence from the above labeling pattern, the graph C_{2k} admits SFDL graph.

Example 3.6: The SFD labeling graph of $\boldsymbol{c_8}$ is given in fig. 1

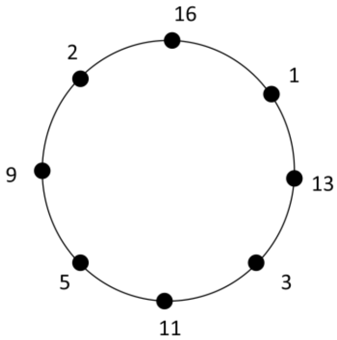

Fig. 3

Theorem 3.7: For each m, n≥ 1, the complete bipartite graph $\boldsymbol{K_{m,n}}$ is SFDL graph.

Proof: Let the bipartition of $K_{m,n}$ be (X, Y) where X = $\{x_1, x_2, \dots . x_m\}$and Y = $\{y_1, y_2, \dots . y_n\}$. Assume that, m ≤ n. But $m \neq n \neq 1$. Then, a super felicitous labeling of $K_{m,n}$ is

f(x_i) = i, $\quad 1 \leq i \leq m$

f(y_1) = mn+m+n

f(y_i) =$f(y_{i-1}) - (m + 1), \quad 2 \leq i \leq n$

Cleary the labels V(G) ∪ E(G) are distinct. Hence from the above labeling pattern, the graph $k_{m,n}$ admits SFDL graph.

Example 3.8: The SFD labeling graph of $\boldsymbol{K_{2,4}}$ is given in fig 2 respectively.

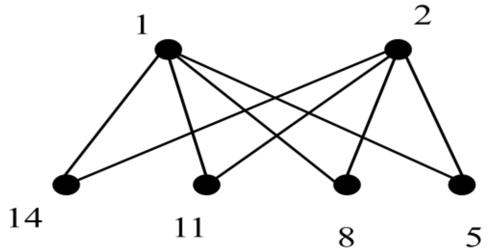

Fig. 4

Theorem 3.9: For a fan graph, $f_n$is SFDL graph for all n ≥ 2.

Proof: Let u_0 be the centre of fan. Define f: V $(f_n) \rightarrow \{1,2, \dots . .3n - 2,3n\}$ as follows:

Case – (i): When n is odd.

f(u_0) = 3n

f(u_1) = 1

f(u_{2k+1}) = f(u_{2j+1}) + 3, $\quad 1 \leq k \leq \frac{n-1}{2}, \; 0 \leq j \leq \frac{n-3}{2}$

f(u_2) = 3n – 2

f(u_{2k}) = f$(u_{2j}) - 3, \quad 2 \leq k \leq \frac{n-1}{2}, 1 \leq j \leq \frac{n-3}{2}$

let f^* be the induced edge labeling of f. Then

$f^*(u_0, u_1) = 3n - 1,$

$f^*(u_0, u_{2k+1}) = f^*(u_{2j+1}) - 3, \quad 1 \leq k \leq \frac{n-1}{2}, \; 0 \leq j \leq \frac{n-3}{2}$

$f^*(u_0, u_2) = 2,$

$f^*(u_0, u_{2k}) = f^*(u_{2j}) + 3, \quad 2 \le k \le \frac{n-1}{2},\ 1 \le j \le \frac{n-3}{2}$
$f^*(u_i, u_{i+1}) = 3n - 3i,\ \ 1 \le i \le n-1$
Case – (ii): When n is even.
f(u_0) = 3n
f(u_1) = 1
f(u_{2k+1}) = f(u_{2j+1}) + 3, $\quad 1 \le k \le \frac{n-1}{2},\ 0 \le j \le \frac{n-3}{2}$
f(u_2) = 3n – 2
f(u_{2k}) = f(u_{2j}) − 3, $\quad 2 \le k \le \frac{n-1}{2}, 1 \le j \le \frac{n-3}{2}$
let f^* be the induced edge labeling of f. Then
$f^*(u_0, u_1) = 3n - 1,$
$f^*(u_0, u_{2k+1}) = f^*(u_{2j+1}) - 3, \quad 1 \le k \le \frac{n-2}{2},\ 0 \le j \le \frac{n-4}{2}$
$f^*(u_0, u_2) = 2,$
$f^*(u_0, u_{2k}) = f^*(u_{2j}) + 3, \quad 2 \le k \le \frac{n}{2},\ 1 \le j \le \frac{n-4}{2}$
$f^*(u_i, u_{i+1}) = 3n - 3i,\ \ 1 \le i \le n-1$
Cleary the labels V(G) ∪ E(G) are distinct. Hence from the above labeling pattern, the graph admits f_n SFDL graph.

Example 3.10: The SFD labeling graph of $\boldsymbol{f_5}$ is given in fig 3 respectively.

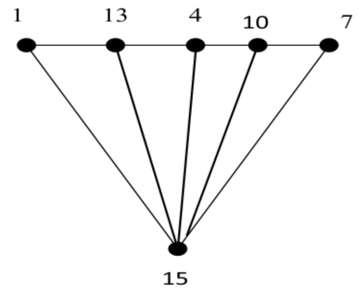

Fig. 5

Theorem 3.11: Any Comb $\boldsymbol{P_n \odot k_1}$ is SFDL graph for all n ≥ 2.
Proof: let $P_n \odot k_1$ be a comb obtained from a path $P_n = u_1, u_2, \ldots . u_n$ by joining a vertex.
Define a function f: V ($P_n \odot k_1$) → {1,2, ...$4n - 3, 4n - 1$} is defined as follows:
f(u_i) = $4n - (2j + 1)$, $\quad 0 \le j \le n - 1$
f(v_i) = 2i+1, $\ \ 0 \le i \le n - 1$
Cleary the labels V(G) ∪ E(G) are distinct. Hence from the above labeling pattern, the graph admits $P_n \odot k_1$ SFDL graph.

Example 3.12: The SFD labeling graph of $\boldsymbol{P_5 \odot k_1}$ is given in fig 4 respectively.

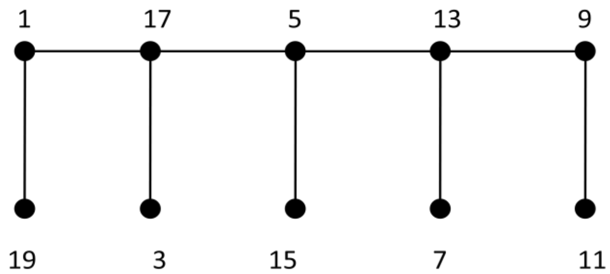

Fig. 6

Theorem 3.13: The graph $\boldsymbol{B_{n,n}}$ is a SFDL graph for all n ≥ 2.
Proof:Let $B_{n,n} = (V, E)$ where $V(B_{n,n}) = \{u, v, [(u_i, v_i): 1 \le i \le n]\}$ and
$E(B_{n,n}) = \{(uv) \cup [(uu_i) \cup (vv_i): 1 \le i \le n]\}$.
Define a function f: V ($B_{n,n}$) → {1,2, ...$4n + 1, 4n + 3$} is defined as follows:
$f(x) = 1$

$f(y) = f(u_n) - 2$
f(u_i) = $4n + 3 - 2j$, $1 \leq i \leq n$, $0 \leq j \leq n-1$,
f(v_i) = i+1, $1 \leq i \leq n$
let f^* be the induced edge labeling of f. Then
$f^*(u, v) = 2n + 2$,
$f^*(u, u_i) = 4n + 2 - 2j$, $0 \leq j \leq n - 1$
$f^*(v, v_i) = 2n + 2 - i$, $1 \leq i \leq n$
Cleary the labels V(G) ∪ E(G) are distinct. Hence from the above labeling pattern, the graph admits $B_{n,n}$ SFDL graph.
Example 3.14: The SFD labeling graph of $\boldsymbol{B_{4,4}}$ is given in fig 5 respectively.

Fig. 7

Theorem 3.15: The Star graph **($\boldsymbol{K_{1,n}}$)** is a SFDL graph for all n .
Proof:Let $K_{1,n} = (V, E)$ where $V(K_{1,n}) = \{u, u_i: 1 \leq i \leq n\}$ and
$E(K_{1,n}) = \{(uu_i: 1 \leq i \leq n\}$.
Define a function f: V ($K_{1,n}$) → $\{1,2, \ldots ..2n - 1, 2n + 1\}$ is defined as follows:
$f(u) = 1$
f(u_i) = $2n + 1 - 2j$, $0 \leq j \leq n - 1$,
let f^* be the induced edge labeling of f. Then
$f^*(u, u_i) = 2n - 2j$, $0 \leq j \leq n - 1$
Cleary the labels V(G) ∪ E(G) are distinct. Hence from the above labeling pattern, the graph $K_{1,n}$admits SFDL graph.

Example 3.16: The SFD labeling graph of $\boldsymbol{K_{1,5}}$ is given in fig 6 respectively.

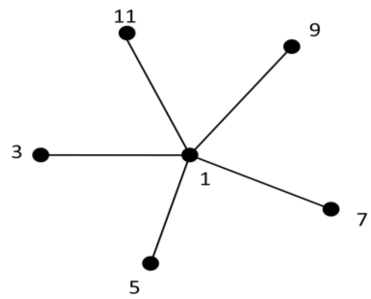

Fig. 8

Results 3.17: Every Friendship graph is not SFDL Graph.
Results 3.18: All ladder (n ≥ 2) graph is FDL graph and NFDL graph but not SFDL graph.

Conclusion:

The Study of labeled graph is important due to its diversified applications. All graphs are not SFDL graphs. It is very interesting to investigate graphs which admit SFD Labeling. In this paper, we proved that Path, Cycle, Complete Bipartite Graph, Fan, Comb, $B_{n,n}$ and Star graph are Super Felicitous Difference Labeling Graph. We have already investigated graphs which are SFDL graph only for certain cases and have planned to investigate the SFD labeling of some special cases of path and cycle related graphs in our next paper.

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