Investigation of peristaltic motion of viscous fluid in a porous channel with Suction and Injection by using Numerical technique. (Galerkin method)

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Abstract

The peristaltic transport of a viscous fluid in an exceedingly porous channel with suction and injection by using Galerkin method is studied. Mathematical modeling was performed with an extended wavelength approximation and a low Reynolds number. The analytical solution is obtained for the velocity field, pressure gradient, friction force within the reference wave frame. The pressure rise and also the frictional force over one wavelength are obtained. The effect of various parameters on pumping characteristics and frictional forces are analyzed graphically. It is observed that the pressure rise decreases because the permeability parameter increases and also increases because the amplitude increases.

Keywords: Viscous fluid, Suction and Injection, Galerkin method.

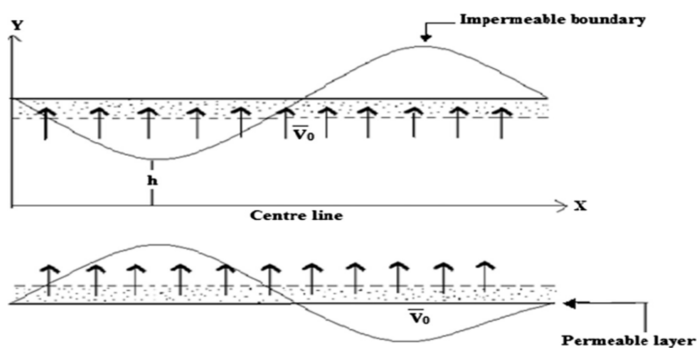
Introduction

The study of peristaltic pumping has received considerable attention in recent decades thanks to its importance in both biological and mechanical situations. Peristalsis consists of the narrowing and transverse shortening of a little of the tube then relaxes, while the lower portion shortens and contracts. Some biomedical tools are manufactured supported the principles of peristaltic pumping. Peristalsis is now well-known to physiologists as a crucial fluid transport mechanism in many biological systems. Specifically this mechanism is involved in urine transport from kidney to bladder, movement of ovum within the fallopian tubes, in movement of chyme within the Gastro intestinal tract, within the transport of spermatozoa, within the efferent duct of the male reproductive tracts, within the duct, within the movement of the ovum, within the female fallopian tubes, in transport of lymph, within the lymphatic vessels and within the vasomotion of small blood vessels. An in depth review on peristalsis is presented by Jaffrin and Shapiro [1]. Tang and Fung [2] investigated longitudinal dispersion of particles within the blood flowing in a very pulmonary alveolar sheet .

Mishra M, Ramachandra Rao A. [3] investigated Peristaltic transport in a channel with a porous peripheral layer, model of flow in a gastrointestinal tract . Pandey S.K. and Dharmendra Tripathi [4] studied force field on the peristaltic flow of a viscous fluid through a finite-length cylindrical tube. Chakradhar K, et.al [5] investigated peristaltic pumping of a micropolar fluid in a tube with permeable wall. Usha S, Sreenadh S, Arunachalam P.V.[6] have studied Peristaltic transport of two immiscible viscous fluids between two permeable walls. In the area of constant flow of incompressible viscous fluid over infinite porous plates subject to suction or injection. Many authors have studied various aspects of the matter, Nandagopal.et.al. [7] have discussed Couette flow of a Bingham fluid in a channel bounded by permeable beds with suction and injection. Mansutti D, Pontrelli G, Rajagopal KR.[8] discussed the Steady flows of Non Newtonian fluids past a porous plate with suction and injection. Chakradhar K, et.al [9] investigated peristaltic pumping of a micropolar fluid in a tube with permeable wall, Pontrelli G, Bhatnagar RK.[10] have studied the flow of a visco elastic fluid between two rotating circular cylinders subject to suction and injection. Majdalani J, Zhou C.[11] had Moderate-to-large injection and suction driven channel flows with contracting or expanding walls.Many bio fluid flows in psychological systems and blood flow in small blood vessels are reported under the mechanism of peristalsis with suction and injection. Nandagopal et.al.[12] studied Peristaltic motion of pseudoplastic fluid in an Inclined channel bounded by Permeable walls, Reddappa et.al. [13] discussed peristaltic transport of conducting williamson fluid in a porous channel Visible of the many physiological applications it is necessary to review the peristaltic transport of a viscous fluid during a channel between porous walls with suction and injection. Anil Kumar, Aggarwal SP [14] approaches Galerkin method for viscous incompressible fluid flow through a Porous Medium in Coaxial Cylinders. Masakazu Shibahara, S.N.Atluri[15] approaches local Petrov-Galerkin method for the analysis of heat conduction due to moving heat source in welding. Ahamad B, Khan, Ahamad S[16] used Galerkin finite element analysis for peristaltic flow of micro polar fluid through porous soaked inclined tube independent of wavelength. In this paper the peristaltic flow of a viscous fluid in a channel with suction and injection through Galerkin method is investigated, under long wave length and low Reynolds number assumptions. The fluid is injected into the channel perpendicular to the lower porous layer with constant velocity V_0 and is sucked bent to the upper permeable layer with the identical velocity V_0, the speed, the pressure rise and friction forces are obtained. The results are deduced and discussed.

2. Mathematical formulation

Consider the peristaltic motion of a viscous fluid in a porous channel of half width a. A longitudinal wave train of progressive sinusoidal waves occurs on the upper and lower permeable walls of the channel. The fluid with a constant velocity V_0 is injected into the channel perpendicular to the lower permeable wall and is sucked out of the upper permeable wall with the same velocity V_0. For simplicity, we restrict our description to the half width of the channel as shown in Fig. 1.



The wall deformation is given by

$$H(X,t)=a+b\sin\frac{2\pi}{\lambda}(x-ct) \tag{1}$$

where b is the amplitude, λ is the wavelength and c is the wave speed. Under the assumptions that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed laboratory frame (X, Y). The transformation between these two frames is given by

$$x = X - ct,\ \ y = Y,\ u(x, y) = U(X - ct, Y),\ v(x, y) = V(X - ct, Y) \qquad (2)$$

We use the following non-dimensional quantities

$$\bar{X} = \frac{x}{\lambda},\ \bar{Y} = \frac{y}{a},\ \bar{t} = \frac{ct}{\lambda},\ \bar{h} = \frac{H}{\lambda},\ \bar{\psi} = \frac{\psi}{ac},\ \bar{\phi} = \frac{b}{a},\ \bar{Q}_1 = \frac{Q_1}{c},\ \bar{v}_0 = \frac{v_0}{c},\ \bar{U} = \frac{u}{c}$$

$$\beta = \frac{\sqrt{k}}{am},\quad \mathrm{Re} = \frac{ac}{\rho},\ \ \sigma = \frac{a}{\sqrt{k}},\ \ \bar{p} = \frac{a^2}{\lambda c \mu} p \qquad (3)$$

where Re is Reynolds number, ϕ is the amplitude ratio, a is the permeability (including slip) parameter, and k is the suction parameter. The equations governing the motions in non dimensional form are

$$\frac{\partial^2 u}{\partial y^2} - k\frac{\partial u}{\partial y} = P \qquad (4)$$

Where $k = \mathrm{Re}V_0$, and $P = \frac{\partial p}{\partial x},\ Q_1 = \frac{P}{\sigma^2}$ (Darcy's law) (5)

The non-dimensional boundary conditions are

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0. \qquad (6)$$

$$u = -1 - \beta\frac{\partial u}{\partial y} \quad \text{at} \quad y = h \qquad (7)$$

3. Solution

Solving Eq. (4) using Galerkin method with the boundary conditions (6) and (7), we obtain the velocity as

$$u = -1 - a_2(y^2 - h^2 - 2\beta\, h) - a_3(y^3 - h^3 - 3\beta\, h^2) \qquad (8)$$

Where $a_2 = P\left(\frac{B_1C_2 - B_2C_1}{A_1B_2 - A_2B_1}\right)$ $\quad a_3 = P\left(\frac{A_2C_1 - A_1C_2}{A_1B_2 - A_2B_1}\right)$ and

$$A_1 = \frac{kh^4}{2} + \frac{(6k\beta - 4)h^3}{3} - 4\beta h^2,\ B_1 = \frac{2kh^5}{5} + \frac{(4k\beta - 3)h^4}{2} - 6\beta h^3,\ \ C_1 = 2\beta h^2 + \frac{2h^3}{3},$$

$$A_2 = \frac{3kh^5}{5} + \frac{(6k\beta - 3)h^4}{2} - 6\beta h^3,\ B_2 = \frac{kh^6}{2} + \frac{(15k\beta - 9)h^5}{5} - 9\beta h^4,\ C_2 = 3\beta h^3 + \frac{3h^4}{4}$$

The volume flux q through each cross section in the wave frame is given by

$$q = \int_0^h u\,dy = -h + a_2(2\beta h^2 + \frac{2h^3}{3}) + a_3(3\beta h^3 + \frac{3h^4}{4}) \qquad (9)$$

The instantaneous volume flow rate $Q(X, t)$ in the laboratory frame between the central line and the wall is

$$Q(X,t)=\int_0^H U(X,Y,t)dY = a_2(2\beta h^2+\frac{2h^3}{3})+a_3(3\beta h^3+\frac{3h^4}{4}) \qquad (10)$$

From Eq. (9) we have,

$$\frac{dP}{dx}=\frac{(q+h)(A_1B_2-A_2B_1)}{2h^2(\frac{h}{3}+\beta)(B_1C_2-B_2C_1)+3h^3(\frac{h}{4}+\beta)(A_2C_1-A_1C_2)} \qquad (11)$$

Averaging Eq. (10) over one period yields the time mean flow (time-averaged volume flow rate) $\bar{Q}$ as

$$\bar{Q}=\frac{1}{T}\int_0^T Qdt = q+1 \qquad (12)$$

4. The pumping characteristics

Integrating Eq. (11) with respect to x over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta P=\int_0^1 \frac{(q+h)(A_1B_2-A_2B_1)}{2h^2(\frac{h}{3}+\beta)(B_1C_2-B_2C_1)+3h^3(\frac{h}{4}+\beta)(A_2C_1-A_1C_2)}dx \qquad (13)$$

The time averaged flux at zero pressure rise is denoted by $\bar{Q}_0$ and the pressure rise required to produce zero flow rate is denoted by ΔP_0 The dimensionless friction force F at the wall across one wavelength is given by

$$F=\int_0^1 h(-\frac{dP}{dx})dx=-\int_0^1 \frac{h(q+h)(A_1B_2-A_2B_1)}{2h^2(\frac{h}{3}+\beta)(B_1C_2-B_2C_1)+3h^3(\frac{h}{4}+\beta)(A_2C_1-A_1C_2)}dx \qquad (14)$$

5. Results and discussion

We have presented the graphical result of pressure rise ΔP and frictional force F with respective time average volume flow rate $\overline{Q}$ for different values of amplitude ratio ϕ, permeability parameter β, suction parameter k, pressure rise ΔP.

In fig 2, the pressure rise ΔP presented with respective time average flow rate $\overline{Q}$ increases as amplitude ratio ϕ increases with fixed values of permeability parameter β, suction parameter k

In fig 3, the pressure rise ΔP with respective time average flow rate $\overline{Q}$ decreases as of suction parameter k increases with fixed values of amplitude ratio ϕ, permeability parameter β

In fig 4, we observed that the pressure rise ΔP with respective time average flow rate $\overline{Q}$ decreases as permeability parameter 'β' increases with fixed values of amplitude ratio ϕ.

suction parameter k.

Frictional force F

From Eq. (14), frictional force F is calculated and from Figs. 5– 7, it is observed that the frictional force shows opposite behavior compared to the pressure rise.

In fig.5, the Frictional force F with respective time average flow rate $\overline{Q}$ increases as of amplitude ratio ϕ increases with constant values of permeability parameter β, suction parameter k.

In fig.6, the Frictional force F with respective time average flow rate $\overline{Q}$ increases some stage then decreases as suction parameter k increases with fixed values of amplitude ratio ϕ, permeability parameter β,

In fig 7,the Frictional force F with respective time average flow rate $\overline{Q}$ decreases as permeability parameter β increases with fixed values of amplitude ratio ϕ, suction parameter k.

Velocity Distribution.

The velocity of the flow is found to change more or less with the variation of the flow parameters. The effect of the flow parameters on the velocity distribution is analyzed with the help of figures (8)-(10) Fig. (8) illustrates the velocity distribution against y for several values of the amplitude ratios. It is found that the increase of the amplitude ratio increase the velocity of the flow field. This is due to the fact that as the fluid viscosity increases, the fluid in both regions of the channel becomes thicker and hence the flow velocity increases. Fig. (9) depicts the effect of the permeable parameter on the velocity distribution. It is observed that the increase in the permeable parameter decreases to the velocity of the flow field . Fig. (10) illustrates the velocity distribution against y for several values of the suction parameter k . It is found that increase of the suction parameter decreases to the velocity of the flow field.

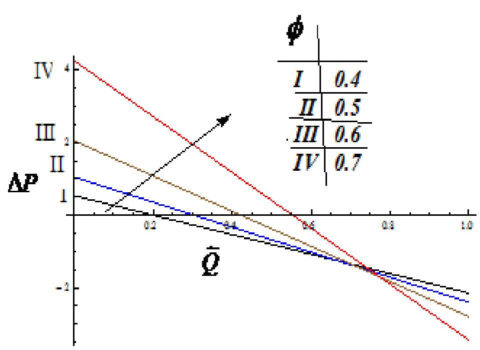


Fig 2. The variation of pressure rise ΔP with time average volume flow rate $\overline{Q}$ for different values of ϕ with fixed β =0.2, k =0.1

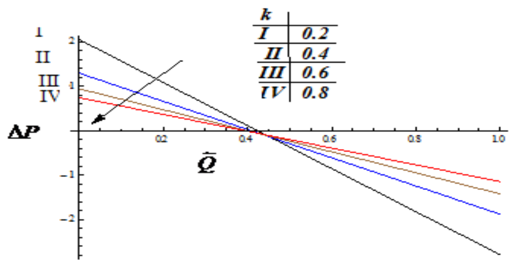


Fig 3.The variation of pressure rise ΔP with time average volume flow rate $\overline{Q}$ for different values of k with fixed ϕ =0.6, β =0.2

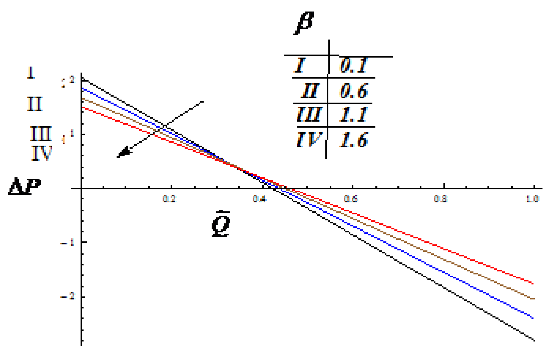


Fig 4.The variation of pressure rise ΔP with time average volume flow rate $\overline{Q}$ for different values of β with fixed ϕ =0.6, k =0.1

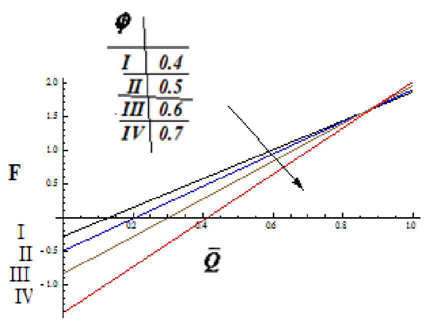


Fig 5.The variation of Frictional force F with time average volume flow rate $\overline{Q}$ for different values of ϕ with fixed β =0.2, k =0.1

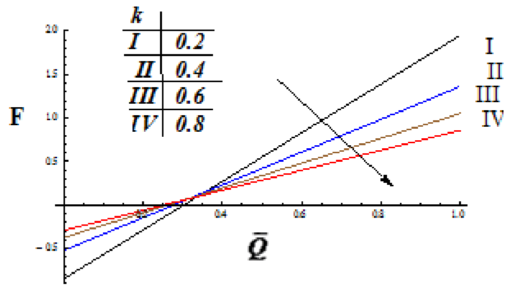


Fig 6.The variation of Frictional force F with time average volume flow rate $\overline{Q}$ for different values of k with fixed β =0.2, ϕ =0.6

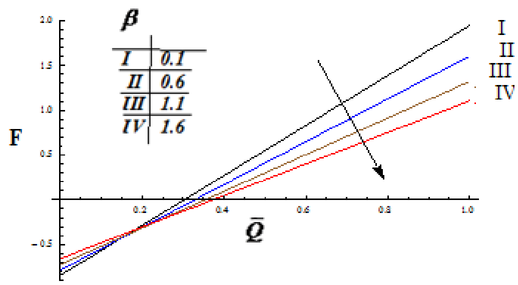


Fig 7.The variation of Frictional force F with time average volume flow rate $\overline{Q}$ for different values of β with fixed k =0.1, ϕ =0.6

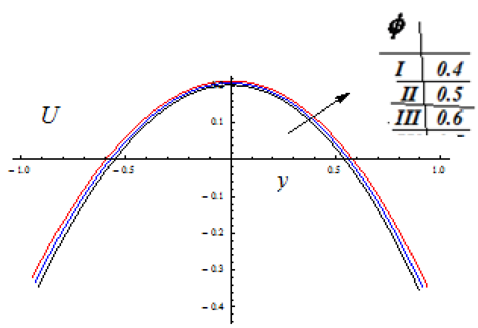


Fig 8. The variation of velocity profile U with different values of ϕ with fixed k =0.1, β =0.2, Q =0.9

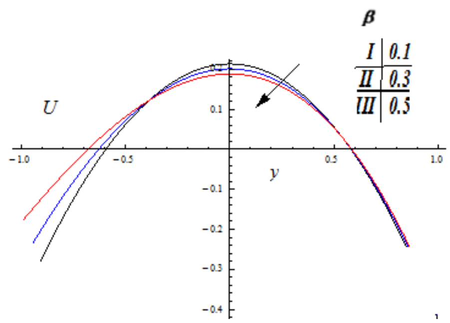


Fig 9.The variation of velocity profile U with different values of β with fixed k =0.1, ϕ =0.6 , Q =0.9

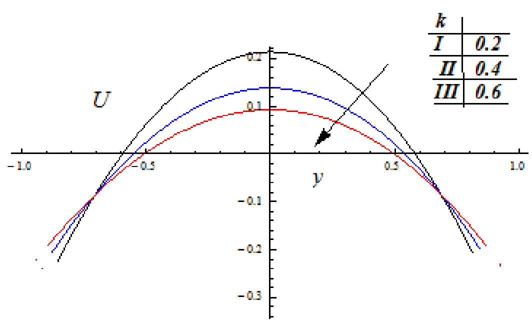


Fig 10.The variation of velocity profile U with different values of k with fixed β =0.2, ϕ =0.6, Q =0.9

6. Conclusions

The Peristaltic transport of a viscous fluid in a channel with suction and injection has been studied in the present work under the assumption of long wavelength and low Reynolds number approximations. The expressions for velocity field, pressure rise and frictional force are determined. It is observed that decreases the pressure rise. as increase in the suction/injection parameter k, Increase in the amplitude ratio ϕ, increases the pressure rise. increase in the permeability parameter β, decreases the pressure rise and the frictional force shows opposite behavior to that of pressure rise with variations k . ϕ and β.

References

[1] Jaffrin MY, Shapiro AH. Peristaltic pumping. Annu Rev Fluid Mech 1971(3):13–36
[2] Fung MY, Tang HT. Longitudinal dispersion of particles in the blood flowing in a pulmonary alveolar sheet. J Appl Mech 1975;42:536–40.
[3] Mishra M, Ramachandra Rao A Peristaltic transport in a channel with a porous peripheral layer: Model of a flow in gastrointestinal tractm 2005;Journal of Biomechanics 38(4):779-89
[4] Pandey SK and Dharmendra Tripathi Influence of magnetic field on the peristaltic flow of a 2010 7(3)169– 176.

[5] Chakradhar K, Nandagopal K, Lakshminarayana P, Sreenadh S, Peristaltic pumping of a micropolar fluid in a tube with permeable wall, International Journal of Mathematical archive 2015; 6 (3) 173– 183.
[6] Usha S, Sreenadh S, Arunachalam PV. Peristaltic transport of two immiscible viscous fluids between two permeable walls, Math Modell Non-linear Syst 1999; 2 385–93.
[7] Nandagopal K,Sreenadh S, LakshmiNarayana P, chakradhar K, Couette flow of a Bingham fluid in a channel bounded by permeable beds with suction and injection. International Journal of Engineering Sciences and Research Technology, 2014; 3 (9) 300 - 305.
[8] Mansutti D, Pontrelli G, Rajagopal KR. Steady flows of non Newtonian fluids past a Porous plate with suction or injection. Int J Numer Methods 1993; 17 927–41.
[9] Chakradhar K, Nandagopal K, Lakshminarayana P, Sreenadh S, peristaltic pumping of a micropolar fluid in a tube with permeable wall, International Journal of Mathematical archive, 2015; 6 (3) 173-183.
[10] Pontrelli G, Bhatnagar RK. Flow of a viscoelastic fluid between two rotating circular cylinders subject to suction or injection. Int. J. Numer. Methods 1997; 24 337–49.
[11] Majdalani J, Zhou C. Moderate-to-large injection and suction driven channel flows with expanding or contracting walls. J Appl Math 2003; 3 181–96.
[12] Nandagopal K, Chakradhar K, Nagender Ch, Sastry TVAP, Peristaltic motion of **pseudoplastic fluid in an Inclined channel bounded by Permeable walls, Int.J. of all** research education and scientific methods, 2022; 10 (3) 1312-1325
[13] Reddappa B, Parandhama A, S. Sreenadh , peristaltic transport of conducting williamson fluid in porous channel, Journal of Mathematical computational science, 2020; 10 (2) , 277-288
[14] Anil Kumar, Agrawal SP , Finite Element Galerkin's Approach for Viscous Incompressible Fluid Flow through a Porous Medium in Coaxial Cylinders, Journal of Mathematical Sciences and Applications. 2013**;** 1 (3), 39-42
[15] Masakazu Shibahara, Atluri SN , The meshless local Petrov-Galerkin method for the analysis of heat conduction due to a moving heat source in welding , International Journal of Thermal Sciences. 2011; 50 (6), 984-992.
[16] Ahmad B., Khan S.U., Ahmad S, Galerkin finite element analysis for peristaltic flow of micropolar fluid through porous soaked inclined tube independent of wavelength , 2021 ARCHIV Proceedings of the Institution of Mechanical Engineers Part E .Journal of Process Mechanical Engineering.