Hypothetical Learning Trajectory of Limit and Derivative Based on Realistic Mathematics Education

Khairudin*, Ahmad Fauzan, Armiati
Doctoral Program of Education Science, Postgraduate – Universitas Negeri Padang (UNP), Indonesia
Corresponding author: ahmadfauzan@fmipa.unp.ac.id

Abstract

Purpose: The learning trajectory of limits and derivatives found in Calculus textbooks is still conventional and complicated for students. Moreover, it does not involve students to discover the concept. This study developed a Hypothetical Learning Trajectory (HLT) based on Realistic Mathematics Education (RME) for Limit and Derivative with a valid and practical GeoGebra.

Methods: This research used design research that combined the Plomp model and the Gravemeijer& Cobb model. There were three phases, the first was preliminary research of analyzing the needs, curriculum, concept analysis, and finding the literature review. In the second phase, the development or prototyping design was conducted through formative evaluations of self-evaluation, expert validation, and one-to-one evaluation. The last phase was assessment or retrospective analysis where evaluating the learning trajectory implementation was performed.

Results: The research results indicate that the HLT has been fulfilled for being a valid learning method to learn limits and derivatives based on the expert judgments and the evaluation result after it has been implemented instructional either in a one-to-one test or in a small group.

Conclusions: The availability of RME-based LIT on Limit and Derivative materials that meet valid criteria with characteristics, namely student-centered learning activities, and the time provided is sufficient to achieve learning goals.

***Keywords**:* HLT, limit and derivative, RME, GeoGebra, mathematical educations

1. Introduction

Studies on Calculus learning find students' difficulty, particularly in understanding the limits and derivatives material (Khairudin, 2020), and the lecturers' failure to recognize the inability of the students to understand the concept of differential calculus (Moru et al., 2014). Furthermore, the learning trajectory of the Calculus tends to be conducted mechanistically; explaining the definitions and concepts, providing several examples of the algorithm, and assigning students to do the exercises. Lecturers rarely elaborate the rationale behind the concepts and algorithms to the students. They merely require students to remember the techniques and methods to solve problems rather than stimulate students to construct and elaborate the concept. As a result, what they have gained in the learning is meaningless and washed away easily (Plomp, 2013).

Study material in Calculus is abstract and full of symbols. Understanding a concept is preferably equipped with understanding other concepts first. Therefore, lecturers should teach calculus from an easy to difficult concept at times. In addition, teachers are also required to elaborate on the concept of Calculus through a real-life situation example and strongly suggested using any media and aids in their instruction. Informing the role of Calculus in learning functions, limits, derivatives, and integrals in various fields are essential. These basic concepts must be comprehended to be able to implement Calculus in various fields. Limits and Derivatives are the core of Calculus. They are the most important part of Calculus to be immediately comprehended.

They are demandingly required in analytical courses such as vector analysis, complex analysis, mathematical-statistical analysis, and advanced calculus. The concept of the limit initiates the concept of derivative and integral. However, to master them, the concept of real numbers and functions needs to be acknowledged beforehand.

The lack of students' understanding of these concepts builds the assumption among students that Calculus is difficult and boring as a result their attitude towards Calculus will also be negative and discourage them to try to construct and elaborate on the concept to solve the math problems (Sumarmo, 2014). In addition, learning is still a teacher- center process that lacks students' involvement. The HTL focuses on predicting what will happen if the students are taught by the designed learning trajectory.

Meanwhile, Realistic Mathematics Education (RME) is a theoretical approach to understanding mathematical concepts through students' everyday experiences. The focus of RME is that students can reinvent mathematics on their own but still under the guidance of adults (teachers/lecturers). In other words, doing contextual problem-solving activities makes it possible for students to reinvent Mathematics. In Indonesia, Realistic Mathematics Education (RME) is known as the Indonesian Realistic Mathematics Approach (PMRI), which originates from the Freudenthal Institute. According to Freudenthal (1991), mathematics can be defined as a human activity. Mathematics should not be placed as a finished product but as an activity or process. Thus mathematics is given to students not in the form of finished products that are ready to use but as a form of activity in constructing concepts in mathematics. RME includes views on mathematics, how students should learn mathematics, and how mathematics should be taught. Instead of having students as the receiver of ready-made mathematics, students should be an active participant who is directed to use situations to reinvent mathematics by using various strategies they have.

The problems used can be derived from various situations (contexts) and they have meaningful contexts that can be related to the material. Concepts in mathematics are acquired through a mathematization process that is initiated by completing the context (context-link solution) to develop students' mathematical understanding more abstractly. Various models applied in the student learning activities can lead to challenging interactions in the classroom that can make students think mathematically. In addition, there is another prominent opinion about learning mathematics that is in learning mathematics students should be provided with realistic situations and the learning should emphasize student activity to allow students to learn mathematics actively. According to Simon (1995), RME has five characteristics, namely: 1) uses contexts; 2) uses models; 3) uses students' own production and constructions; 4) interactive teaching process; and 5) intertwined various learning strands.

These characteristics can be described as follows: 1) Based on the "real world" context. The use of RME learning begins with the use of contextual problems. It must be by the concepts to be studied. It is also referred to as conceptual mathematization. By utilizing the formalization process and its abstraction power, students develop concepts and apply them to other fields (applied mathematization). Mathematization of everyday experience and applying mathematics in students' lives are important to note so that mathematical concepts follow students' daily lives; 2) The use of models (mathematization) is related to the situation made by the students themselves (self-developed models). Self-developed models play a role in leading students to change a real situation to more abstract situations (informal mathematics to formal mathematics). A Step that students can take is to design situations that are close to the students into several models, which later will turn into models-of through a higher reasoning process, the model-of will move into a for-model of various similar problems. This process later will give creation to a formal mathematical model; 3) Use of production and construction. By producing and constructing themselves students are motivated to reflect on everything that arises in the learning process. Informal strategies that students create in contextual problem solving will be an inspiration for them in constructing more formal mathematical knowledge and developing further learning; 4) Create interaction. In RME creating student interaction with teachers is important. Interactions that occur can be questions, explanations, arguments over agreeing or disagreeing, as well as reflection. This form of interaction is used by students during the process of forming and developing informal mathematics into formal mathematics; and 5) Intertwinement. In RME the interrelationship between materials in mathematics is essential. Learning does not place attention on relatedness with other fields, which will affect problem-solving. The application of mathematics requires knowledge related to other and more complex fields, not just arithmetic, algebra, or geometry.

The learning trajectory was first developed by Simon (1995) who stated that there are three main components of HLT;1) Learning goals; 2) Learning activities; and 3) Hypothetical learning process. Before Clements & Sarama (2004) developed it into; 1). Learning objectives; 2) Progress in the development of thinking and learning; and 3) Stages of instructional tasks. Followed by Andrews-Larson (2017) who added aspect to 4). The role of instructors (Lecturers) in supporting the development of mathematics learners at each stage of the activity. Simon & Tzur (2004) provides four principles that must be considered in building an HLT: 1) Build HLT based on the understanding of current learners; 2) HLT is a tool for planning the learning of specific mathematical concepts; 3) Instructional tasks or worksheets as a tool for improving learning on mathematical content are key to the learning process; and 4) Instructors must modify every aspect of HLT continuously due to the nature of hypothetical processes associated with uncertainty.

Simon & Tzur (2004) outlined the mechanisms for the development of mathematical concepts based on aspects of constructivism, i.e. constructs of assimilation based on Piaget's theory, (1971, 1980). Piaget insists that learners cannot simply accept new concepts. He insists learners must build new concepts through assimilation by accommodating previous concepts. Based on the problems that have been raised and because of the importance of improving learning flow, especially RME-based Limit, and Derivative materials, the purpose of this research is how to design Learning Instructional Trajectory (LIT) material Limit and Derivative material that is valid and practical.

2. Methods

HLT or LIT was developed through *design research* proposed by Gravemeijer and Cobb (2006); Plomp, 2013). To implement HLT, it is necessary to design material modules containing teaching objectives that are tailored to the activities carried out by students, predictions or comments, anticipation or completion of each activity carried out by students, and time allocation.

The LIT development is begun with a thought experiment, namely thinking about the Hypothetical Learning Trajectory (HLT). Then, the reflection on the experiment's results should be done. After that, the experiment is continued with the next thought experiment. In the long-term period, thought experiments are always together as a result the LIT with a high level of resistance could be obtained. Gravemeijer & Cobb's development design has three steps as follows: 1) preliminary research (preparing the experiment); 2) prototype phase (Stage of Development and assessment); and analyzed retrospectively. In the research formulation, several data collection techniques are used in collecting research data, including questionnaires, checklists, and interviews. The instrument used is adjusted to the data collection technique, as shown in Table 1 below.

Table 1. Data collection techniques and research instruments

No	Research activities	Data collection technique	Aim	Research instruments
I	Preliminary research			
1	Needs Analysis	Interview with lecturer, and students	Collecting information from lecturers and students so that HLT is developed according to needs	Interview guide for lecturers and students
2	Curriculum Analysis	Checklist	Seeing the essential concepts to support the achievement of the Limit and Derivative concept	Checklist
3	Literature Review	Checklist	Seeing the accuracy of presenting the HLT concept and the accuracy of integrating RME principles in the module	Checklist
II, III	Stages of development (validity and practicality test) and assessment			
1	Self-evaluation	Validation	Seeing the validity of the product by the researchers themselves	Self-evaluation validation sheet
2	Expert Validation	Validation	See the feasibility of the content, presentation, graphics, language of the developed product	HLT validation sheet and Module validation sheet
3	One-to-one experiment	Questionnaire, and Interview	Obtain data on the practicality of the product developed and the results of interviews	Student practicality questionnaire sheet, and interview guidelines

The data collection techniques used in this study included questionnaires, checklists, and interviews under the research steps, The instrument used was adjusted to the stages. The data analysis technique used was qualitative data processing by describing the criteria for validity and LIT's practicality.

1. Data analysis at the Preliminary research: The data obtained in the preliminary research were interviews and observation, then the analysis technique used was descriptive methods by data reduction, data presentation, and conclusions drawing.

2. Validity Data Analysis: Data analysis for the HLT validity test used the percentage analysis technique based on Khairudin et al (2021) with Formula: $Validity\ scores = \frac{total\ score\ obtaind}{maximum\ score}\ x\ 100\%$ with criteria such as Table 2 below.

Table 2. Product validity criteria

Percentage earned	Category
<51	Invalid
51-70	Quite valid
71-90	Valid
91-100	Very Valid

3. Practical Data Analysis: Practical analysis, according to Khairudin et al (2021), the analysis is done by providing student response questionnaires after learning and calculating scores using the following Formula: Practical scores $= \frac{total\ score\ obtaind}{the\ total\ score\ of\ the\ number\ of\ students}\ x\ 100\%$, with criteria such as Table 3 below.

Table 3. Product validity criteria

Percentage earned	Category
90-100	Very practical
80-89	Practical
65-79	Practical enough
55-64	Less Practical
<54	Not Practical

3. Results/Findins and Discussion

3.1. Preliminary research results

3.1.1. Needs analysis results

Based on interviews with lecturers who teach Limits and Derivatives, the information obtained was that the material was perceived as a difficult material to teach to students. The difficulty leads to inadequate learning outcomes obtained. Limits and Derivatives is the basic material in calculus, and this content makes students learn how to think logically, which is the basic object for studying advanced Calculus and analytical courses. Limits and Derivatives are also possible to be designed according to the characteristics of students, close to students' lives, and make students discover the concepts of Limits and Derivatives for themselves. In addition, the current learning trajectory used by the lecturer so far has not paid attention to the overall characteristics of students. The learning aids used also have not considered the student learning trajectory. Therefore, the available learning tools of material in textbooks and lecturers' instructions do not facilitate problem-solving abilities and student learning outcomes. On the other hand, interviews with students stated that they liked the content which related to their everyday life and used visual material and computer simulation animation.

3.1.2. **Curriculum Analysis Results**

The curriculum used in the Bung Hatta University Mathematics Education study program has referred to the Industrial Revolution 4.0 Curriculum and the MBKM curriculum. Limit and Derivative of Calculus course are by Course Learning Outcomes (CPMK) and Student Learning Outcomes (CPL). From the results of the curriculum analysis, it was found that the Limits and Derivatives learning content did not contain daily life and problems experienced by the students. Thus, it is necessary to teach the topic of Limits and Derivatives starting from things close to students' lives to find out for themselves the concepts contained in the Limits and Derivatives. So, it is necessary to design the learning for Limits and Derivatives that can guide students in understanding the concept well by involving the problems they often experience in their daily life.

Based on the analysis of CPMK and CPL in the curriculum used at Bung Hatta University Mathematics Education, the Limit and Derivative material begin with function. Besides, the concept of limit is introduced numerically by calculating the values of a function near a point with the help of Excel. Furthermore, the concept of derivative begins with determining the gradient of the tangent to a function. For this reason, changes were made in the initial presentation of the concept of limit by presenting cases in various fields. It is necessary to separate the concepts of Limit and Derivatives, although Limits support the concept of Derivatives. Concept Analysis Results The subject of Limits and Derivatives begins with the material of functions, and the concept of limits is introduced numerically by calculating the values of a function near a point with the help of Excel. Furthermore, the concept of derivative begins with determining the gradient of the tangent to a function. For this reason, changes were made to the initial presentation of the concept of limit by presenting cases in various fields. It is necessary to separate the concepts of Limit and Derivatives, although Limits support the concept of Derivatives. Concept Analysis Results The subject of Limits and Derivatives begins with the material of functions, and the concept of limits is introduced numerically by calculating the values of a function near a point with the help of Excel. Furthermore, the concept of Derivatives begins by determining the gradient of tangent lines on a function. For this reason, it is made a change in the initial presentation of the concept of limits by bringing up cases in various fields. It is necessary to separate the concept of Limit and Derivative, although the concept of Derivative is supported by Limit.

3.1.3. Concept analysis results

The first concept that students must master is the concept of a function that does not have a function value at a certain point along its definition area, for example $f(x) = (1 + x)^{\frac{1}{x}}$ and $f(x) = x^x$, function that has no value at $x = 0$. When the value$f(0)$cannot be determined, it is traced around that point, either from the left or right of the point. It helps students find the concept of "Limit". With the help of GeoGebra, it turns out that the value of the function limit can be determined with $\lim_{x\to 0} f(x) = \lim_{x\to 0}(1 + x)^{\frac{1}{x}} = e$, while $\lim_{x\to 0} x^x$ there is none. It gave birth to the concept of a limit at a point. Likewise, the concept of limit at infinity is found $\lim_{x\to -\infty}(1 + x)^{\frac{1}{x}} = -\infty$ and $\lim_{x\to +\infty}(1 + x)^{\frac{1}{x}} = 1$. Next, students learn the distance function, which is written as$s = s(t)$, which states the length of the path traveled during the travel time. Based on the formula for speed, as$v = \frac{s}{t}$and when the distance traveled varies with the time taken, of course, the speed of the object changes. It gives the concept of the average speed for the distance traveled from time t_0 - t_1 with the formula$\bar{v} = \frac{s(t_1)-s(t_0)}{t_1-t_0}$. Suppose the travel time is very small to 0, or very close to (can also be written$t_1 \rightarrow t_0$).So that the speed at which time $t_0$can be determined by $\lim_{t_1 \to t_0} \bar{v} = \lim_{t_1 \to t_0} \frac{s(t_1)-s(t_0)}{t_1-t_0} = s'(t_0)$ what is better known as the Derivative function s at the $t_0$point's. This form is also found in the inflation rate, population growth rate, and the tangent line gradient (Geometry). Based on this definition, it is possible to determine the derivative functions for various functions and develop them in Derivative Function with $f'(x)$formulas.

3.1.4. Literature review

Some of the research that has been done to develop LIT include, Fauzan et al (2018) designing the LT for Teaching Social Arithmetic using the RME Approach, Sari & Julianti (2018), designing LT for understanding the concept of angles, Brophy & Lowe (2017) creating LT to develop computing and programming capabilities, Swidan (2019) make LT for the Fundamental Theorem of Calculus material, Meika et al (2019) make LT for combination material, Fauzan & Sari (2017) make LT for understanding the concept of Fractions, Tamba et al (2018) designing LT for quadratic inequalities, Bisognin et al (2019) for Curve Length, Syafriandi et al (2018) as well as Campos & Fontanar (2019) for Statistics material, Nusantara & Putri (2018) for the slope of the line, Clements et al (2019) for geometric shapes, Czarnocha (2016) for Linear equations, Rahayu & Wijaya (2018) designing LT for Statistical thinking and finally Khairudin et al (2020) has made HLT for Green's Theorem material. Especially in Limit and Derivative material, HLT based on RME has not been developed.

3.2. Prototyping phase results

Based on the findings of the preliminary study, HLT was designed for RME- based on Limit and Derivative materials which were divided into three phases, namely, prototype design, formative evaluation, and revising prototype phase.

3.2.1. Prototype design Hypothetical Learning Trajectory (HLT)

Prototype one was designed based on the results of preliminary research analysis and the stage of preparing for the experiment. There are 2 (two) goals that will be achieved after limit and derivative learning, namely, first, students can find the concept of Limit and Derivative. Learning objectives are applied to the instructional flow and elaborated into learning activities based on RME learning characteristics. The instructional flow design can be seen in Fig 1 below.

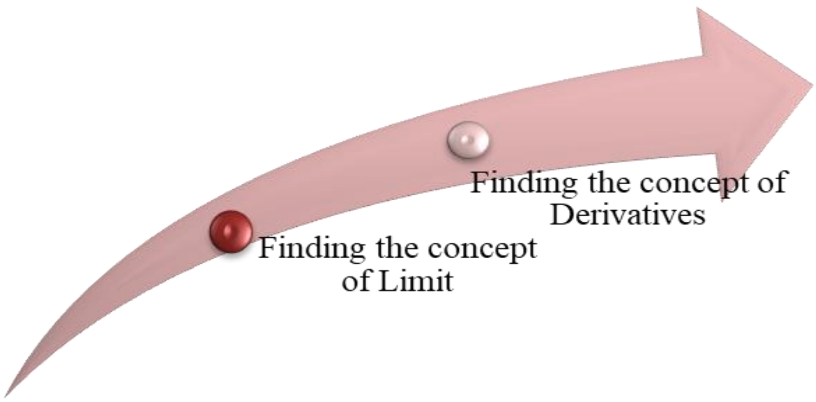

Fig 1.Instructional flow design of limit and derivative concepts

HLT is designed to be implemented in the learning process in the form of student modules. Two goals will be achieved after limit and derivative learning, namely, students can find the concept of Limit and Derivatives. Learning objectives are applied to the instructional flow and elaborated into learning activities based on RME learning characteristics. The learning flow of finding the Limit concept can be seen in Fig 2 below.

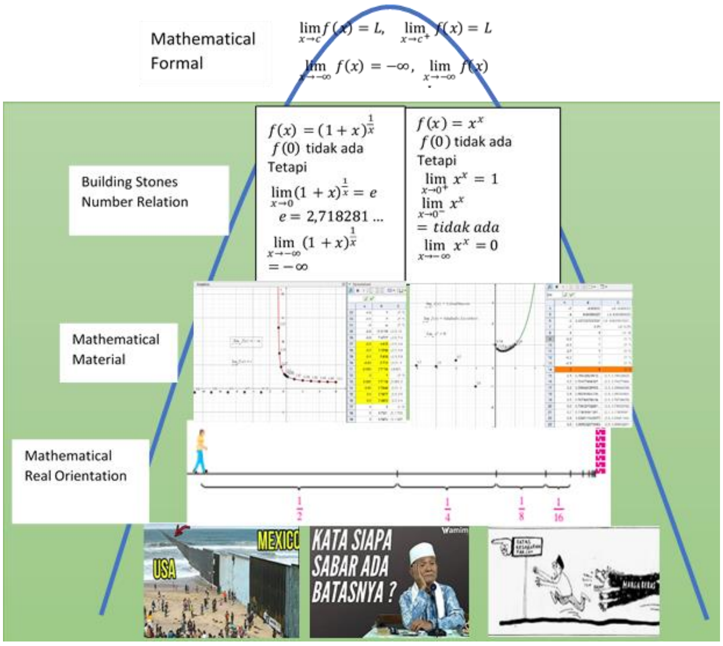

Fig 2. Learning trajectory finding the concept of limit

In Fig 2, 3 (three) activities are designed, students' thinking hypotheses consist of predicting student answers, lecturers' anticipation, how students' answers, and student thinking processes are included in the LIT book. Formally according to diagram 1 of the learning flow process, students find several limit concepts, including; $\lim_{x\to c} f(x) = L$, and $\lim_{x\to c^+} f(x) = L$ $\lim_{x\to c^-} f(x) = L$ $\lim_{x\to c^+} f(x) = \pm\infty$ $\lim_{x\to c^-} f(x) = \pm\infty$ $\lim_{x\to -\infty} f(x) = L$ $\lim_{x\to +\infty} f(x) = L$. Meanwhile, the learning flow to find the concept of Derivatives can be seen in diagram 2. In diagram 2,the formal mathematical concepts of Derivatives will be obtained and found at one point. Based on diagram 2, from the process of learning flow, students find the concept of Derivative, namely and.$f'(c) = \lim_{\Delta x\to 0} \frac{f(c+\Delta x)-f(c)}{\Delta x}$ $f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$. In each learning path, orientation is given to the problems encountered in their daily lives.

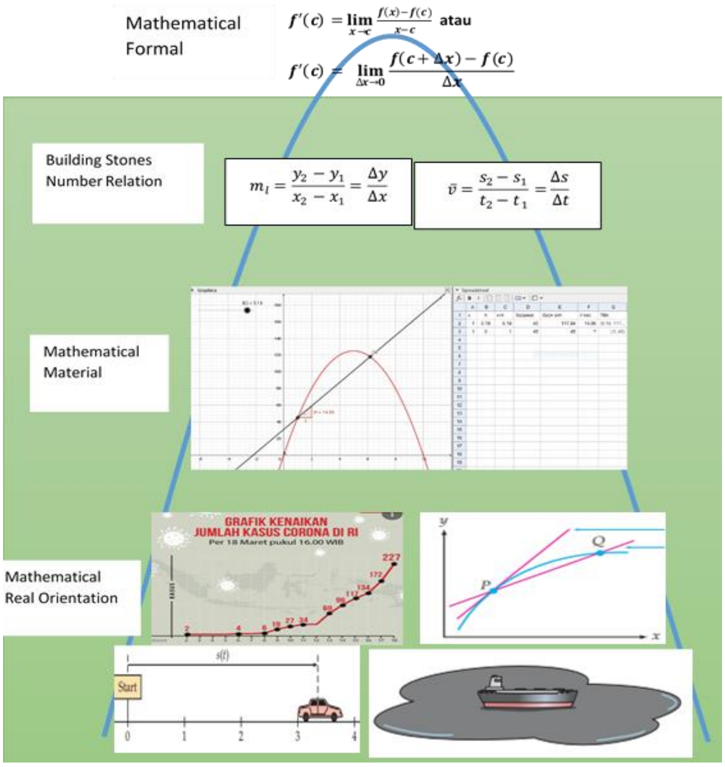

Fig 3. Learning trajectory finding the concept of derivatives

3.2.2. Formative Evaluation Results

Formative evaluation is carried out to assess the quality of the product design results developed. A formative evaluation was used to assess the quality of the HLT design results and student modules, developed by Tessmer (2013); Plomp (2013) namely self-evaluation, expert validation, one-to-one evaluation, and small group because only to test the practicality of products on a small scale.

Self evaluation results: The first formative evaluation carried out was a self-evaluation, in which the researcher looked at the suitability of the HLT and the student module that had been designed with RME-based HLT indicators. In this self-evaluation, many changes occurred in the design of the HLT and student modules. It was especially in terms of the appearance of the HLT book and its size. Consideration of researchers revising the module's presentation to make it attractive and meet the needs of students. In addition, from the results of this self-evaluation, errors also occurred in typing errors, sentence structure, and punctuation errors, for example, a question mark left in the interrogative sentence.

Validation results by experts: After improving the self-evaluation stage, the process was continued by the validation process done by the experts on HLT constructs, content, and modules. The validators were two Mathematics Education lecturers familiar with teaching Limits and Derivatives. Aspects that are assessed are LIT content and language. The content aspect consists of 11 indicators, namely student-centeredness, use of context, goal-oriented process, use of models (mathematization), Interactiveness, completeness of HLT elements (goals, assignments, and hypotheses), the accuracy of goals, the accuracy of contextual problems, the accuracy of activities facilitating vertical mathematization and horizontally, the accuracy of activities to find concepts, and the accuracy of time allocation. While the language aspect consists of 4 indicators, namely, the language used is easy for students to understand, the accuracy of the choice of words, the effectiveness of the sentences, and the sentences used do not have a double meaning. The complete results of HLT validation are shown in Table 4 below.

Table 4, Expert validation results

No	Indicator	V1	V2	Total score	Max Score	%	Criteria
1	The designed HLT is student-centred	5	5	10	10	100	Very valid
2	Contextual problems in each activity are appropriate to introduce the concept of Limits and Derivatives based on RME	5	4	9	10	90	Valid
3	The problems presented support the achievement of learning objectives	5	5	10	10	100	Very Valid
4	The problem stimulate students to make their own models (self-develop models)	4	5	9	10	90	Valid
5	Existing activities encourage students to interact in learning	5	4	9	10	90	Valid
6	The HLT component, which includes learning objectives, a set of tasks to achieve the goals and the hypothesis of how students learn and the anticipation of the lecturer, is complete	5	5	10	10	100	Very Valid
7	The learning objectives described in the sub-goals are correct	4	5	9	10	90	valid
8	The activities given to achieve the goals are appropriate.	5	4	9	10	90	valid
9	The activities of each meeting are appropriate to direct students to horizontal mathematization and vertical mathematization.	4	5	9	10	90	valid
10	Activities are appropriate to bring students to rediscover the concept of Limits and Derivatives (guided reinvention)	5	5	10	10	100	Very valid
11	Precise and systematic HLT sequence	5	4	9	10	90	valid
12	The time allocation designed is right for each meeting	4	4	8	10	80	valid
13	The language used is easy for students to understand	5	5	10	10	100	Very valid
14	The word chosen is correct	5	5	10	10	100	Very valid
15	The sentence used is effective	5	5	10	10	100	Very valid
16	The sentences used are easy to understand and do not cause double meaning	5	5	10	10	100	Very valid
	Amount	76	75	151	160	94.38	Very Valid

The results obtained from the two aspects obtained from the instrument filled with the validator with a total score of 151, with a maximum score of 160, so that the validation score is 94.38% with valid criteria. From these results, for each validated aspect item, there are very valid and valid results. The HLT product is feasible to be used in learning with a time of 2 meetings. The validation results are complemented by several comments from the validator for LIT improvement. It includes sharpening the concept of limit, which is not just determining the value of a function at a point. However, it was around a point, and this has been followed up in implementation.

One-to-one evaluation results: For one-to-one evaluation, a limited practicality test of the product, the HLT implemented in the student module was tested on three students taking the Differential Calculus course with categories of students with low, medium, and high abilities. Researchers directly observed the online activities of the three students according to the time allocation. Students carried out instructions starting from activity 1 with instructions: "As a student, of course, you are familiar with many terms or use of words" Limit, try to mention any problem in which we often hear and find the term Limit or what is similar to the word limit in our everyday life." The results of the worksheets made by students are shown in Fig 4 below.

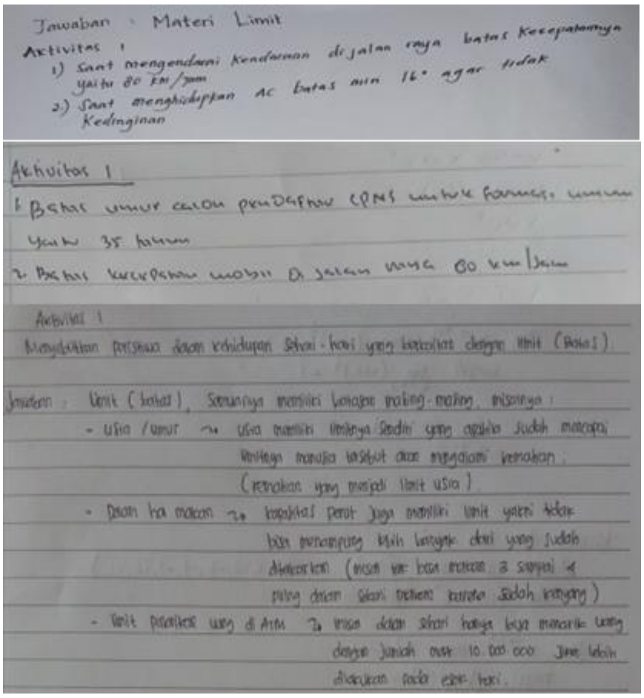

Fig 4. Student worksheets in activity 1.

After that, the students didthe activity 2 with the instruction "Now, please investigate the GeoGebra application visually which contains graphs and spreadsheets to investigate the behavior of the given function values according to the given module, try to determine the area of the actual function definition and the points where the function does not have a value of f(x). and its surroundings say x≠0. All students could work on the results, as shown in figure 3. In inactivity 2, students found only positive values according to the graph. However, after investigating in more detail, students could find the actual value of the function definition area with answers according to the objectives in activity 2, that is: For the function$f(x) = (1+x)^{\frac{1}{x}}$, we get the definition area is all real numbers x > 0, plus negative odd integers because the value of the function also exists for$x = -1, -3, -5, -7, \ldots$, which turns out to be dots (discrete) as indicated by the points in x < 0. for the function$f(x) = x^x$, the definition area is also x> 0, plus a negative integer $x = -1, -2, -3, -4, \ldots$indicated by the dotted image on the graph.

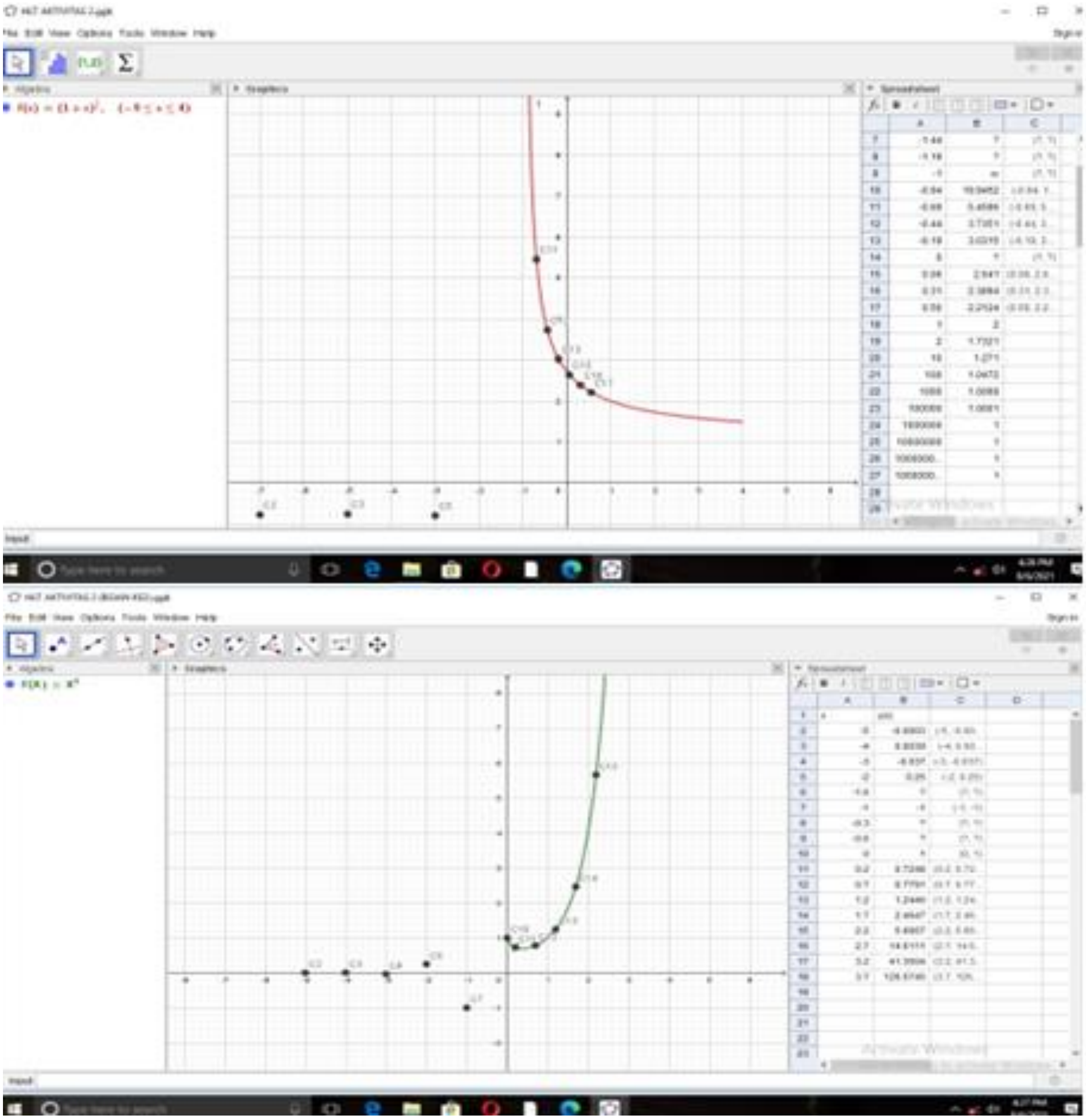

Fig 5. Results of activity two on GeoGebra

Then, in the activity 3 with the instructions: "Now try to conclude how the condition of the function value when the value of x goes to a point where there is no function value or goes to an infinite value to the far left and right, Use the symbol $\rightarrow$ (arrow) for the target number. When $x \rightarrow ?$ Then $f(x) \rightarrow ?$ (read: when x goes to a certain number, then it goes to a certain number). The results of student answers can be shown in Fig 6 below.

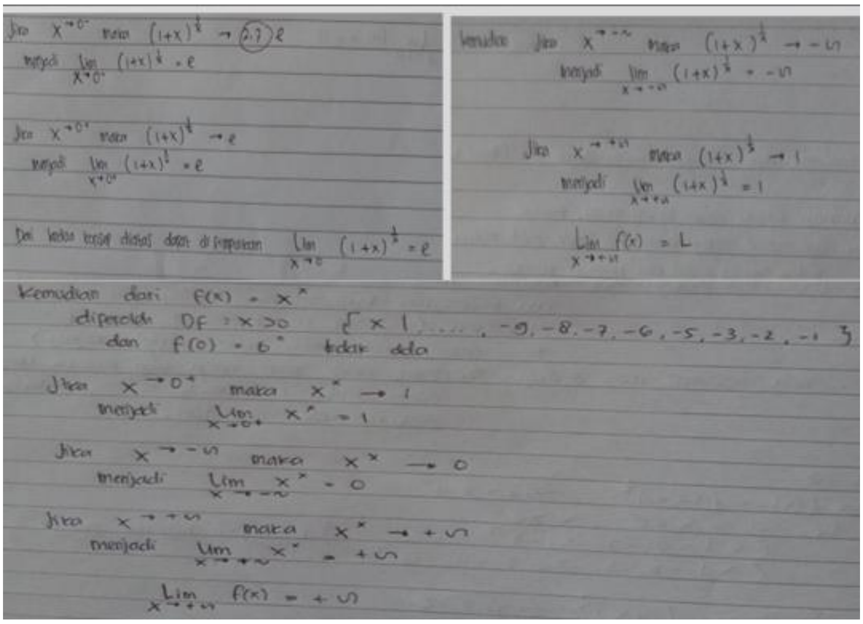

Fig 7. The results of activity three on student worksheets

In learning objective 2, which is to find the concept of Derivatives with 3 (three) activities, namely: 1) Students mention some problems that have been studied related to the rate of change or the speed of a quantity; 2) Students determine the average rate of a quantity at a certain time interval if the function is known and the time interval is small to 0; and 3) Students conclude the concept of Derivatives with Mathematical symbols. For activity 1, almost all students could mention several examples in everyday life about the concept of rate, including; Motor speed, population growth rate, inflation rate, swimmer rate, and others. However, in activity 2, there were several obstacles in creating programs in GeoGebra to visualize and animate the motion of points on a tangent line. However, the lecturer anticipated that visualization and animation of function graphs and their derivatives simultaneously appeared, as shown in Fig 7 below.

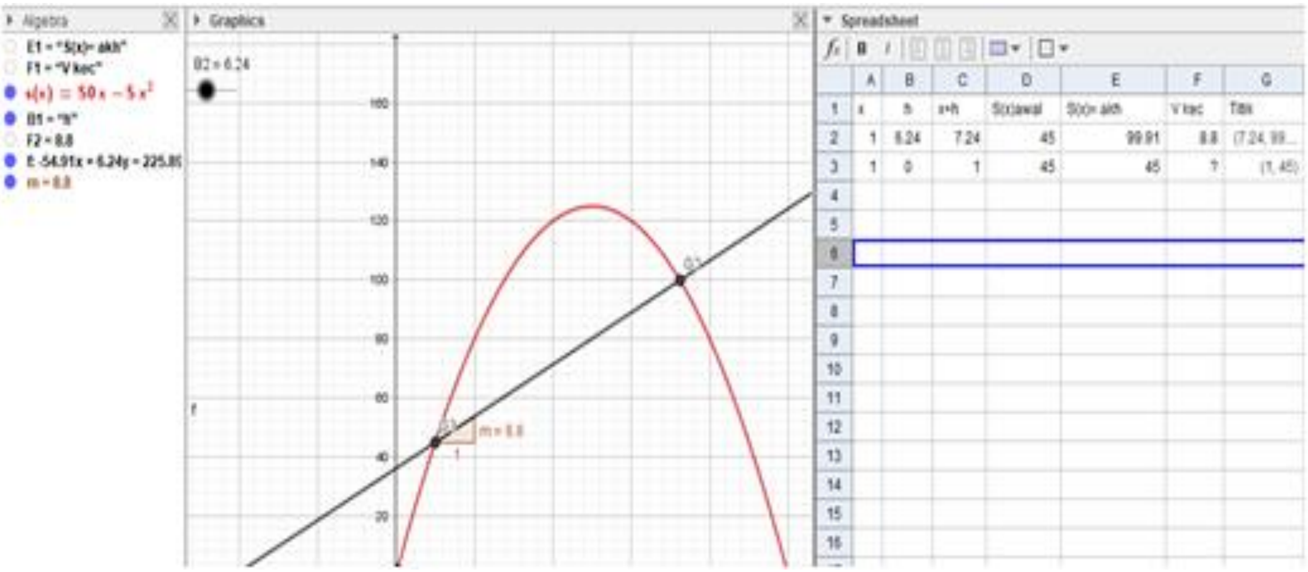

Fig 8. The results of activity 2 in the GeoGebra program with the concept of derivatives

Finally, Activity 6 could be reached with the conclusions made by students, namely:$f'(c) = \lim_{\Delta x \to 0} \frac{f(c+\Delta x)-f(c)}{\Delta x}$Or. $f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$. In general, students could carry out all activities well with constraints on programming in GeoGebra. Besides, the lecturers could anticipate so that learning objectives could be achieved.

RME-based LIT practical results: A questionnaire was immediately given to test the LIT practicality for RME-based Limits and Derivatives in the one-to-one implementation. Practicality questionnaires were given to students after participating in the lesson. The results of the practical analysis of student modules can be seen in Table 5 below.

Table 5. Recapitulation of practicality questionnaire results

No	Indicator	S1	S2	S3	Total score	Max Score	Percentage	Criteria
I. Attractiveness/everyday contextual								
1	The problems given in the module are related to problems in everyday life	4	5	4	13	15	86.67	Practical
II. Usage process								
2	The use of images in the module helps students understand the problem given	5	4	5	14	15	93.33	Very Practical
3	The use of pictures in the module leads students to find the concept of Limit and Derivative material	5	4	4	13	15	86.67	Practical
4	This module helps students understand the concept of Limits and Derivatives	5	5	4	14	15	93.33	Very practical
5	The use of modules does not create a boring learning atmosphere	5	4	4	13	15	86.67	Practical
III. Ease of Use								
6	Using the module makes it easier to find the concept of Limit and Derivative material	5	4	4	13	15	86.67	Practical
7	The use of modules makes it easier to achieve learning objectives.	5	5	4	14	15	93.33	Very practical
8	The use of the module makes it easier to understand the concept of Limit and Derivative material	5	5	4	14	15	93.33	very practical
9	The use of modules makes it easier to develop reasoning and creative abilities	4	4	4	12	15	80.00	Practical
10	The activity steps in the module are easy to understand.	4	5	4	13	15	86.67	practical
IV. Time								
11	The use of the module is under the available time allocation.	5	5	4	14	15	93.33	very practical
	Amount	52	50	45	147	165	89.09	Practical

Based on Table 5, it can be concluded that the implementation of the HLT product is going well and can be done by students with a practical value of 89.09%. On the other hand, based on the results of interviews with students, the students reported that they were excited in learning using computer applications, such as GeoGebra. With graphic visualization and numeration, and animations, it is easier for students to understand the concepts of Limits and Derivatives, which are difficult to understand if they study from textbooks or teaching materials provided by the lecturer.

3.3. Retrospective analysis

After the learning process ended, the researcher looked back at the implementation of LIT through video recordings. The activity at meeting 1, the learning objective of finding the concept of limit, provides a stimulus for students to find the concept of limit through activities to identify problems from events involving the word limit in everyday life. In all these activities, generally, all students can determine and name the requested event or object to find the form of limit at one point, Limit at Infinity and limit to infinity at infinity. Lecturers as facilitators provide guidance and direction to students who have difficulties. In this study, each student has a different way from his friends of mentioning the RME context.

This study aims at developing LIT through activities that develop students' thinking and creative abilities to discover and understand complex and abstract Calculus concepts. This study used a combination of two development designs, namely the Plomp development design and the Gravemeijer and Cobb development design[3], although for the practicality test to the one-to-one evaluation step. The learning design of the Gravemeijer & Cobb model was used to develop HLT to LIT, and for its operation, is in the form of student modules. The first phase carried out in the preparation of HLT into LIT is preliminary research. Based on preliminary research data, students need an engaging guide or module, making it easier for them to find concepts, starting from problems close to their daily lives. Preliminary research also reveals that for Limits and Derivatives, learning requires a LIT by the level of students' thinking abilities, a LIT appropriate to the context of life experienced by students. Meanwhile, concept analysis and literature review support the establishment of LIT. The initial design of the RME-based HLT was based on the results of preliminary research and RME principles. In this research, the activities are arranged in HLT, taking into account the principles of RME. Five principles are considered: the actual context close to students and the relationship between previous mathematical material. Early studies of learning HLT have focused broadly on learning mathematics topics in elementary schools, such as measuring lengths, fractions, line gradients, combinations, and linear equations. Learning trajectory research has been carried out on more advanced topics such as algebraic reasoning, calculus, and trigonometry in recent years. Weber & Thompson (2014) have developed a hypothetical learning trajectory to generalize students' understanding of graphs from a function of one variable to a graph of a function of two variables, guided by Realistic Mathematics Education instructional design theory. Furthermore, Swidan (2019) designed an HLT on Fundamental Theorem Calculus (FTC), built with digital tools, and student interactions were examined using these tools. This study contributes to the literature on calculus learning by following: the learning process through a designed learning trajectory, as students learn in real life.

This HLT for Limits and Derivatives can assist researchers in constructing and investigating different learning trajectories for topics in calculus. The actual learning trajectory design and learning focus identification improve calculus teaching practice. It is argued that using the learning trajectory as a learning tool and applying it in actual educational practice requires further clarification and research. The main results of this study indicate that through the activity of solving contextual problems in the learning path, students can find: 1) the concept of limit at one point; 2) the concept of left and right limits; 3) the concept of limit at infinity; and 4) the concept of infinite limit. This discovery was stimulated by the principles of RME, namely guided reinvention, didactical phenomenology, and emerging models of the characteristics of RME, especially students' free production and students contributions. This study also shows that the learning flow for the topic of Limits and Derivatives developed has a positive impact on students' understanding of concepts with the help of GeoGebra. The initial design of the RME-based HLT was based on the results of preliminary research and RME principles. In this research, the activities were arranged in HLT by taking into account the principles of RME. Five principles were considered: the actual context close to students, paying attention to the relationship between previous mathematical material, allowing students to carry out horizontal and vertical mathematization processes, and allowing interaction between students and lecturers. The limitations of the research on the practicality test are only up to one-to-one evaluation. It is necessary to carry out a large-scale test to get LIT results with a high resistance level.

4. Conclusions

Based on the findings and data obtained in this study, the following conclusions were obtained: 1) The availability of RME-based LIT on Limit and Derivative materials that meet valid criteria with characteristics, namely student-centered learning activities, real problems at the beginning of learning according to the goals of learners. Activities in LIT facilitate students to perform vertical and horizontal mathematical processes, the accuracy of activities in finding concepts assisted by GeoGebra; and 2) LIT based on RME fulfills applicable criteria with the characteristic of LIT being able to run at all levels of the student, assisting students in finding concepts, developing students' thinking skills, and the time provided sufficient to achieve learning goals.

Refrences

[1]. Khairudin, K. (2020). Kemampuan Awal Kalkulus Mahasiswa Pendidikan Matematika. *Edukasi: Jurnal Pendidikan*, *18*(1), 50-60.

[2]. Moru, E. K., Qhobela, M., Wetsi, P., & Nchejane, J. (2014). Teacher knowledge of error analysis in differential calculus. *pythagoras*, *35*(2), 1-10.

[3]. Plomp, T. (2013). Educational design research: An introduction. *Educational design research*, 11-50.

[4]. Sumarmo, U. (2010). Berpikir Dan Disposisi Matematik: Apa, Mengapa, dan bagaimana dikembangkan pada peserta didik. *Bandung: FPMIPA UPI*. 1–27.

[5]. Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for research in mathematics education*, *26*(2), 114-145.

[6]. Clements, D. H., & Sarama, J. (2012). Learning trajectories in mathematics education. In *Hypothetical Learning Trajectories* (pp. 81-90). Routledge.

[7]. Andrews-Larson, C., Wawro, M., & Zandieh, M. (2017). A hypothetical learning trajectory for conceptualizing matrices as linear transformations. *International Journal of Mathematical Education in Science and Technology*, *48*(6), 809-829.

[8]. Simon, M. A., & Tzur, R. (2012). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. In *Hypothetical Learning Trajectories* (pp. 91-104). Routledge.

[9]. Khairudin, K., Fauzan, A., Armiati, A., & Suryani, K. (2021). Analisis Kebutuhan Model Problem Posing Berorientasi STEM. *JSHP: Jurnal Sosial Humaniora dan Pendidikan*, *5*(2), 159-167.

[10]. Fauzan, A., Armiati, A., & Ceria, C. (2018). A learning trajectory for teaching social arithmetic using RME approach. *IOP Conference Series: Materials Science and Engineering*. 335(1), 012121.

[11]. Sari, A., & Julianti, B. (2018). Asessing Hyphotetical Learning Trajectory of Mathematics Teachers. *Journal Of Teaching And Learning In Elementary Education (JTLEE)*, *1*(1), 1-10.

[12]. Brophy, S. B., & Lowe, T. A. (2017). A learning trajectory for developing computational thinking and programming. *ASEE Annu. Conf. Expo. Conf. Proc.*, vol. 2017-June, 2017.

[13]. Swidan, O. (2020). A learning trajectory for the fundamental theorem of calculus using digital tools. *International journal of mathematical education in science and technology*, *51*(4), 542-562.

[14]. Meika, I., Suryadi, D., & Darhim, D. (2019). Developing a Local Instruction Theory for Learning Combinations. *Infinity Journal*, *8*(2), 157-166.

[15]. Fauzan, A., & Sari, O. Y. (2017). Pengembangan Alur Belajar Pecahan Berbasis Realistic Mathematics Education. In *Prosiding Seminar Nasional Pascasarjana Unsyiah*.

[16]. Tamba, K. P., Saragih, M. J., & Listiani, T. (2018). Learning trajectory of quadratic inequality. *JOHME: Journal of Holistic Mathematics Education*, *2*(1), 12-21.

[17]. Bisognin, E., Bisognin, V., & Rodrigues, E. B. (2019). A learning trajectory to the understanding of the curve length concept. *Acta Scientiae. Revista de Ensino de Ciências e Matemática*, *21*(3), 24-40.

[18]. Syafriandi, S., Fauzan, A., Lufri, L., & Armiati, A. (2020). Designing hypothetical learning trajectory for learning the importance of hypothesis testing. *Journal of Physics: Conference Series*. 1554(1), 012045.

[19]. Campos, P. R., & Fontanari, J. F. (2019). Predictability of imitative learning trajectories. *Journal of Statistical Mechanics: Theory and Experiment*, *2019*(1), 013501.

[20]. Nusantara, D. S., & Putri, R. I. I. (2018). Slope of straight line in ladder: A learning trajectory. In *Journal of Physics: Conference Series*. 1097(1), 012116).

[21]. Clements, D. H., Sarama, J., Baroody, A. J., Joswick, C., & Wolfe, C. B. (2019). Evaluating the efficacy of a learning trajectory for early shape composition. *American Educational Research Journal*, *56*(6), 2509-2530.

[22]. Czarnocha, B. (2016). Learning trajectory: Linear equations. In *The Creative Enterprise of Mathematics Teaching Research* (pp. 373-393). Brill.

[23]. Rahayu, N. A., & Wijaya, A. (2018). Hypothetical Learning Trajectory: A Case of Statistical Thinking. *Journal of Physics: Conference Series*. 1097(1), 012109.

[24]. Khairudin, Fauzan, A., Armiati, & Suryani, K. (2020). Developing Hypothetical Learning Trajectory for Green's Theorem. *International Journal of Advanced Science and Technology*, *29*(10s), 1300 - 1309

[25]. Weber, E., & Thompson, P. W. (2014). Students' images of two-variable functions and their graphs. *Educational Studies in Mathematics*, *87*(1), 67-85.