Mutual Value Reflection and Automorphisms

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Abstract: This article discusses how all numerical systems can be self-contained, mutually valuable. If a given set is infinite and no condition is required for it to have a set of mutually valuable reflections, then the number of such reflection is infinitely large.
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Introduction

The concept of function plays an important role in mathematics and its applications. The function is a special case of reflection. The definition and value domains of a function are the same set, and it is easy to study when the function has the same value. If the field of definition and values of a function is a single set and it is finite, it is possible to determine the number of such functions and all its manifestations. If the number of elements in a set is n, then the set is called a substitution of one-valued reflections, and their number is equal to $n!$.
This article discusses how all numerical systems have the same value.

The main findings and results

If a given set is infinite and no condition is required for it to have a set of mutually valuable reflections, then the number of such reflections is infinitely large. However, if one or more algebraic structures (addition, multiplication of elements) are defined in a set, it is necessary to perform the letters corresponding to the algebraic structures in the sets in order to reflect the set's self-value, the number of such reflections is limited, even single.
The following is a set of natural, integer, rational, real, complex numbers that represent each other with the same value.

In the set of *N*-natural numbers two algebraic operations (addition and multiplication) are defined, that is, optional n and m for natural numbers $n+m \in N$ and $n \cdot m \in N$ the relationship is fulfilled.

A set of natural, integer, rational, real, complex numbers, respectively N, Z, Q, R, C let's define it with such letters. The "+" and "x" operations are defined in each of these sets.

Definition. If the set *K* is one of the number systems listed above, or an algebra of quaternions, $f: K \to K$ to reflect a mutual value

$$f(a+b) = f(a) + f(b) \quad (1)$$

$$f(a \cdot b) = f(a) \cdot f(b) \quad (2)$$

any of the conditions $a, b \in K$ if appropriate for the elements, f reflection is called automorphism.

Let's say *K = N*, $f: N \to N$ using the condition of automorphism (2) any $a \in N$ for a natural number $f(a) = f(a \cdot 1) = f(a) \cdot f(1)$ equations can be written. From the last equation and the condition of automorphism (1)

$$f(2) = f(1+1) = f(1) + f(1) = 1+1 = 2$$

namely,

$$f(2) = 2$$

we create an equation. In the same way

$$f(3) = 3, f(4) = 4, \ldots$$

equations can be determined. Of these $f: N \to N$ it can be determined that the automorphism consists of a single exact reflection.

Let's say *K = Z*. Even in this case $f: Z \to Z$ we show that the automorphism consists of the exact reflection, that is, the *Z*-ring has an autorphism consisting only of the exact reflection. Using the condition of automorphism (1), the following equations can be written for an arbitrary *a*-integer:

$$f(a) = f(a+0) = f(a) + f(0) \text{ or } f(0) = 0$$

All other integers are used to reflect the transition to *f* only $f(1) = 1, \ f(-1) = -1$ suffice it to say. The latter equations are derived from the following equations, respectively:

$$f(a) = f(a \cdot 1) = f(a) \cdot f(1) = a \cdot f(1), \ a \in Z$$

$$0 = f(1-1) = f(1+(-1)) = f(1) + f(-1) = 1 + f(-1)$$

Hence, the Z ring also has a unique autorphism.

Let's say, $K = Q$, $f: Q \to Q$ be an arbitrary automorphism. As above, $f(0) = 0, f(1) = 1$ and any whole *f* reflection can be used to show self-reflection. Any for that $\frac{1}{q} \ (q \in N)$ we show that the rational number in the form *f* returns to itself by reflection.

$$1 = f(1) = f\left(q \cdot \frac{1}{q}\right) = f(q) \cdot f\left(\frac{1}{q}\right) = q \cdot f\left(\frac{1}{q}\right)$$

So, $f\left(\frac{1}{q}\right)=\frac{1}{q}$ equality is appropriate. Using this

$$f\left(\frac{p}{q}\right)=f\left(p\cdot\frac{1}{q}\right)=f(p)\cdot f\left(\frac{1}{q}\right)=p\cdot\frac{1}{q}$$

we write the equation. Thus, any rational number $\frac{p}{q}$ for $f\left(\frac{p}{q}\right)=\frac{p}{q}$, f we have proved that automorphism is only an exact substitution.

Let's say, $K=R$, $f:R\to R$ let the reflection satisfy the conditions of autorphism.

Theorem 1. The field of real numbers *R* has a unique autorphism consisting only of exact substitutions.

Proof. Let's say $f:R\to R$ an automorphism. $Q\subset R$ based on the relationship and $f(0)=0,\ f(1)=1$ any rational number of relationships $\frac{p}{q}$ for $f\left(\frac{p}{q}\right)=\frac{p}{q}$ can be proved as above. Any irrational number to prove the theorem completely α f it is enough to prove that it can be regained by reflection. Firstly α is an irrational number $f(\alpha)$ will also be positive. $\alpha>0$ so $\sqrt{\alpha}$ number will be available and $\sqrt{\alpha}\in R$ attitude is appropriate. In that case,

$$f(\sqrt{\alpha})=f(\sqrt{\alpha}\cdot\sqrt{\alpha})=f(\sqrt{\alpha})\cdot f(\sqrt{\alpha})=(f(\sqrt{\alpha}))^2>0$$

attitude is appropriate. So, $f(\alpha)>0$. That's it α and β for real numbers $\alpha<\beta$ if the relationship is fulfilled, $f(\alpha)<f(\beta)$ the attitude is also appropriate. So, *f* the automorphism *R* maintains the order relationship in the field. To complete the proof of the theorem, let us assume the inverse, i.e., autorphism α irrational number β let it be irrational number and $f(\alpha)\neq\alpha$. $\alpha\neq\beta$ or, $\alpha<\beta$, or $\alpha>\beta$ the relationship is fulfilled. Suppose, $\alpha<\beta$ let the inequality be reasonable. Any two irrational numbers in particular α and β There is at least one rational number *r* between, i.e. $\alpha<r<\beta$. Assumption based $\alpha<f(\alpha)$ inequality is appropriate. f because the reflection maintains the order relationship

$$f(\alpha)<f(r)<f(\beta)$$

inequalities are appropriate or

$$\beta<f(r)<f(\beta)$$

But, $f(r)=r$ the equation holds because *r* is a rational number. So $\beta<\alpha$ is done, but the last inequality $r<\beta$ is the opposite of inequality. So, $f(\alpha)=\alpha$ is done. The theorem is proved.

Let's say *K = C*, $f:C\to C$ let it be an automorphism. In this case, the reflection *f* is not unique, because the exact reflection is different $f(\alpha)=\overline{\alpha}$, namely $f(a+bi)=a-bi$ visual reflection is also an automorphism. In addition, the area of complex numbers in [1] has been studied without proof of the existence of an infinite number of automorphisms.

Below we present only two areas of complex numbers, with the additional condition of automorphism

$$f_1(a+bi)=a+bi$$

$$f_2(a+bi)=a-bi$$

show that we have autorphisms.

Theorem 2. If $f:C\to C$ as an automorphism $f(R)=R$ if the equation holds, that is, if any real number *f* passes to the real end again using reflection, then f_1 or, f_2 is the same as reflection.

Proof. $f:C\to C$ is an automorphism, any real number $a\in R$ for $f(a)\in R$ let the relationship be fulfilled. In that case any using Theorem 1 $a\in R$ for a real number $f(a)=a$ we determine the fulfillment of the equation. If $\alpha=a+bi$ if there is an arbitrary complex number,

$$f(a+bi)=f(a)+f(b)f(i)=a+bf(i)$$

$$f(a-bi)=f(a)-f(b)f(i)=a-bf(i)$$

we write the equation. Of these

$$a^2+b^2=f((a+bi)(a+bi))=a^2-b^2(f(i))^2$$

equality arises. and assuming that the arbitrary is a real number

$$a^2+b^2=a^2-b^2(f(i))^2$$

from equality $(f(i))^2=-1$ equality arises. $f(i)\in C$ attitude and ultimate equality $f(i)=i$ or $f(i)=-i$ we create an equation. So, $f=f_1$, or $f_1=f_2$ equality only one of course is fulfilled. The theorem is proved.

Conclusion

Result: If $f:C\to C$ is an automorphism, if it is continuous $f=f_1$ or, $f=f_2$ one of the equations is of course fulfilled.

Proof. If $f:C\to C$ if it is an automorphism, then the rational reflection of any rational number can be proved in the same way by reflection *f*. It is well known that any irrational number can be expressed as the limit of a sequence of rational numbers. α arbitrary irrational number $\{r_n\}_1^\infty$ if, α a sequence of rational numbers approaching, i.e. $\lim_{n\to\infty} r_n=\alpha$. f from the continuity of automorphism $f\left(\lim_{n\to\infty} r_n\right)=f(\alpha)$ equality is appropriate. From this $\lim_{n\to\infty} f(r_n)=\lim_{n\to\infty} r_n$ using the equation,

$$f(\alpha)=f\left(\lim_{n\to\infty} r_n\right)=\lim_{n\to\infty} f(r_n)=\lim_{n\to\infty} r_n=\alpha$$

arises. So, f each using reflection α the real number is reflected again. Hence and according to Theorem 2, $f=f_1$ or $f=f_2$ one of the equations is valid.

References

[1] Nechaev V. I. “Chilovye systems”. Moscow: Enlightenment. 1975.
[2] Nasirov S. N., Sheraliev N. N. “From the experience of student scientific supervision of GTH” Pedagogical education, 2003 №6.
[3] Davronovich, A. D., & Ugli, S. A. A. (2021). About A Certain Class Of Non-Associative Algebra. *International Journal of Progressive Sciences and Technologies*, *25*(2), 234-236.
[4] Aroev, D. (2021, November). ABOUT THE ROOTS OF A POLYNOMIAL WITH A WHOLE COEFFICIENT. In *International Scientific and Current Research Conferences* (pp. 14-16).
[5] Nosirovich, N. S., Davronovich, A. D., & Abdurashid og’li, S. A. (2021). SOME PROPERTIES OF THE DISTANCE BETWEEN TWO POINTS. *Journal of Ethics and Diversity in International Communication*, *1*(1), 54-56.
[6] Aroev, D. (2021, November). ABOUT THE ROOTS OF A POLYNOMIAL WITH A WHOLE COEFFICIENT. In *International Scientific and Current Research Conferences* (pp. 14-16).