A Review on Dynamic Response Modelling of Faulted Gearbox

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Abstract
Gearbox degrades over time due to load, speed, and environmental factors. The faults develop over time. Early detection and understanding of the mechanism of these are essential. The current work is to consolidate the gearbox's dynamic models to understand the gearbox's behavior under faulted conditions.
Keywords: Gear mesh stiffness, gear mesh damping, dynamic response, degree of freedom, force.

1. Introduction

The gearbox forms an integral part of many mechanisms used and provides the size to speed and torque transmission advantage in the system. The gear elements transmit power and rotary motion by successive teeth engagement. Gear transmission has several advantages over the other transmission mechanisms in terms of overall dimensions, operational simplicity, and higher transmission efficiency. The gears' science is continuously developing to enhance transmission life, operating efficiency, and reliability. The gearbox is used in industrial, civilian and military applications, for example, in conveyor belts, wind turbines, and helicopters (mining industry etc.). The gearbox operates under both constant and varying operating conditions. The gearbox consists of gears, shafts, bearings and support structures. Under the varying operating condition, the gears continue to degrade. If the gear faults are not detected early, this may be the reason for the substantial monetary and life losses [1]. In the gears, the stresses are pure rolling at the pitch line; above and below the pitch line, rolling-sliding action takes place; the sliding is in the opposite direction [2–6]. The sliding interfaces have no problem if appropriately lubricated. In insufficient lubrication, surface disparities are in direct contact and differential surface hardness, rise in temperature, and adhesive bonding under high pressure contribute to the breakdown of the gear surfaces[2,5]. The gear's roots have tension (loaded side) and compression (opposite side) simultaneously. The root is the point of highest stress in tension. The bending strength of the root is the direct function of the surface hardness, surface smoothness, sharpness of radius and the fault (crack/ pitting etc.)[2,7]. Under these conditions, the gear material is under continuous degradation and when the failure crosses the threshold called failure. The gear failures are classified based on lubrication and non-lubrication. The gear pairs work under the elastohydrodynamic lubrication and partial-elastohydrodynamic lubrication mechanism. The non-lubricated failures include the overload and bending types of failure. The lubricated failure is a set of fatigue (pitting), wear, and scuffing. The gear teeth, due to deterioration, generate dynamic forces which accelerate the gear tooth failure. In literature, the tribological studies on the failure of gears[8–17] and bearing[18–31] and synthesis of lubricant is carried out to understand the mechanisms. Lubricant properties are determinantal for the safe working of the interfaces in contact. The wear is a continuous process when two surfaces are in contact, and the threshold depends upon the expected lifetime.[2]

Every machine element exhibits a unique vibration signature under standard conditions. The faults change the signature in how it is related to the fault. If a machine does not have any fault, it generates vibrations; these vibrations relate to the periodic events of machine operation. These consists of harmonics of the rotating shaft, meshing of gear teeth, electric field etc. These harmonics are a reliable indicator of the sources and can be used as a diagnostic tool[32,33]. The vibration from different sources consists of different information; to quantify the source, it is necessary to understand the behaviour of the vibration output of the different sources. The vibration analysis consists of studies such as time, frequency, and time-frequency domains [34–36].

All types of wear (pitting, mild-wear, scoring, etc.) affect the gearbox's dynamics and are responsible for the high vibration and noise. The dynamic modelling to mimic this phenomenon of analysis of the contacting mechanism of the gear is developed, and the system's dynamic behaviour under various health conditions can be simulated. The gear meshing phenomenon is highly complex and needs a design-based understanding of the gear system[37–39]. The different fault types can be simulated, revealing fault detection and diagnosis. The present work consolidates the dynamic models of the healthy and faulted gearbox.

2. GEARBOX FAULT MODELLING AND FAULT DIAGNOSIS TECHNIQUES

2.1 Gearbox Dynamic Modelling

2.1.1 Gearbox Modelling (Translation and Torsional Vibration)

All Gearbox is a complex system, and modelling such a complex system requires a better understanding of the physical laws like Newtonian laws of motion (vector analysis), energy laws (Langrangian laws: scalar analysis), and laws of equilibrium to simulate the responses. The simulation provides the advantage of non-interference of the surrounding environment noise. Dynamic modelling summarises the stresses (contact and bending stresses), type of fault (pitting, mild-wear, scoring, and crack), transmission efficiency, loads on machine elements (like a load on bearings), natural frequencies of the system, the response of the system, reliability, life of the component, whirling of rotors[40–44]. The gear system's complexities limit simulation study by restricting that all details are not modelled [45]. If it is tried to model all the details, the governing equations present more non-linearities, which are hard to solve by mathematical operations and increase computation cost [46,47]. Two techniques are used to model the gear train: 1: lumped parameter model, 2. Finite element model. In lumped parameter model, the components are considered with masses concentrated at a set of points. In the finite element model, the masses are distributed, and the final response is the integration of the responses. The initial studies estimated the dynamic load on gears systematically during the 1920s and 1930s. In the 1950s, the simple spring-mass system was presented to estimate the dynamic load; later, the models were expanded to evaluate the dynamic behaviour of gear in the mesh. After the 1970s, more focus has been on estimating gear stiffness, non-linearity of system element, damping, effects of friction in excitation, gear errors (transmission, manufacturing error), a different mode of vibration, steady-state and transient responses of the system[48–56]. The mathematical models are grouped into the following classification: (1) Simple dynamic models: estimation of the dynamic factor to estimate the stress on the tooth (2) Tooth compliance model: these are the single degree of freedom models, and only the tooth stiffness included as the potential energy storing element in the system rest of the elements flexibilities are neglected. (3) Gear dynamic model: include the flexibilities of other elements along with tooth compliance. (4) Geared rotor dynamics model: torsional vibration of the system is included in the model along with whirl, gyroscopic effect, and time-varying mesh stiffness.

Sometimes, it is not easy to separate all these from each other when a gear tooth is in contact with two primary components of the load in respect of transmitted power: 1. Static component, and 2. Dynamic component (due to fluctuating conditions). So, many researchers provide the dynamic factor, defined as the ratio of static load and dynamic load[57,58].

$$Dynamicfactor = \frac{Staticload}{Dynamicload} \quad (1)$$

The concept of speed is introduced at a later stage, and the modified dynamic factor as stated:

$$(Dynamicfactor)_{modifeid} = \frac{600}{(600 + pitchlinevelocity\ (infeetperminute))} \quad (2)$$

Buckingham published a report in 1931 in the ASME research committee on strength of gear teeth. The report suggested that "speed over 5000 fpm, the change in load-carrying capacity of gear negligible, instead depends more on the effective mass, effective error". Buckingham dynamic load factor is given as:

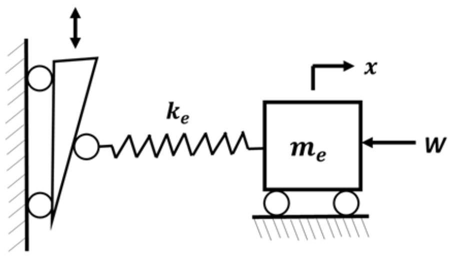
$$F_d = F_t + \frac{21\,v(b \times C + F_t)}{21\,v + \sqrt{b \times C + F_t}} \quad (3)$$

Where Fd is the total load on the gear, including load due to dynamic action, Ft is the maximum tangential load, and C is the load stress factor $\left(C = \frac{k \times e}{\left(\frac{1}{E_p} + \frac{1}{E_g}\right)}\right)$ in N/mm, k is a factor according to the pressure angle of the gear $\left(k = \begin{cases} 0.107 & for 14.4^{\circ} involute full depth \\ 0.111 & for 20^{\circ} involute full depth \\ 0.115 & for 20^{\circ} involute stub \end{cases}\right)$, Ep & E.g. modulus of elasticity of pinion and gear material, and e is the sum of errors between two meshing teeth (mm).

The gear tooth is designed by considering the gear as a cantilever beam. The Lewis formula is used to express the load[59]. The design of the gear tooth influences the dynamic characteristics. The optimization of the design can bring a considerable reduction in the gear vibration amplitudes[60]. The influence of parameters like load, speed, design, friction, and roughness are studied to see the change in dynamic characterization[42,61–71]. The transmission error is one of the criteria for changing the gear condition; the static and dynamic transmission errors are affected by the type of fault [58,72–74]. The transmission error is defined as " the difference between the angle rotated by the pinion and the correspondence angle rotated by gear" [75].

Tables 1 and 2 show the different dynamic models considering the effect of only interaction between the gear teeth and including the effect of the other system elements like motor, shaft, bearings and the load, respectively.

Table 1 Gear dynamic models [76–79]

(a)	 1-Degree of freedom model (Tuplin	$m_e\ddot{x} + k_e(t)x = W$ Where, $k_e(t)$ is equivalent to time-dependent mess stiffness of contacting teeth, m_e equivalent mass, and W excitation force

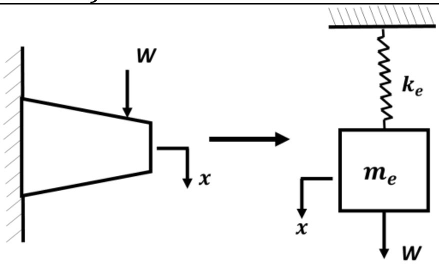
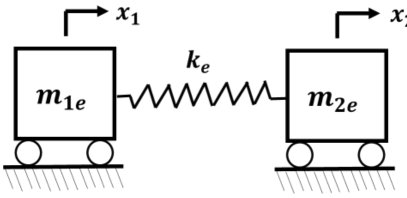
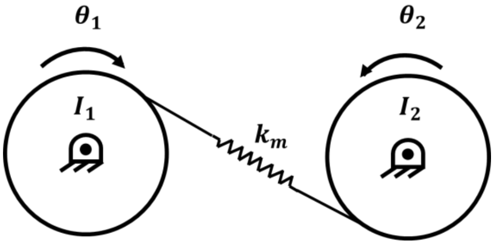
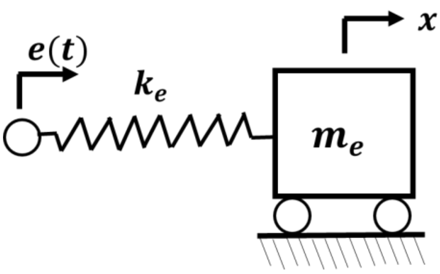
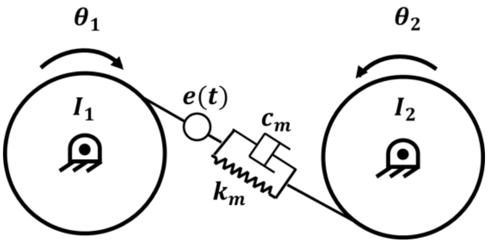
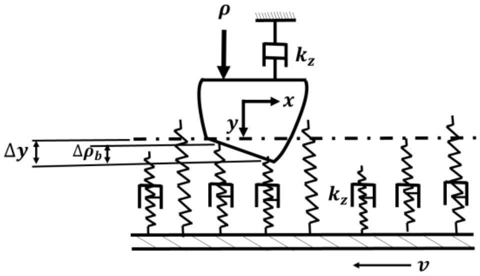
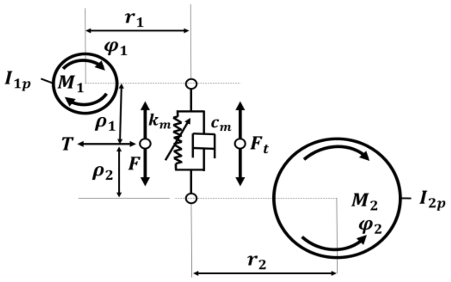
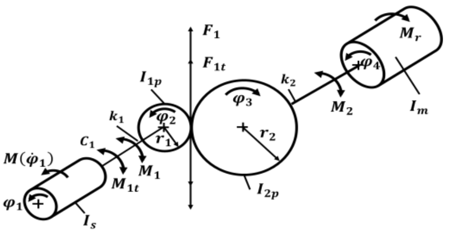
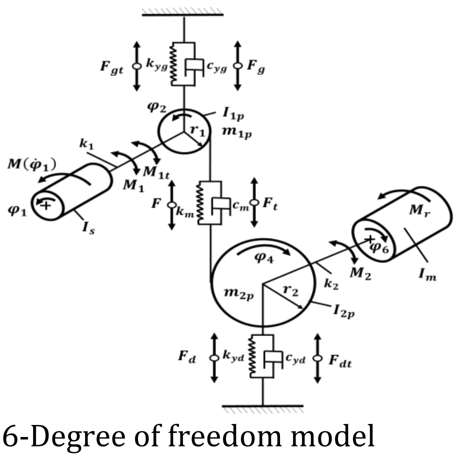
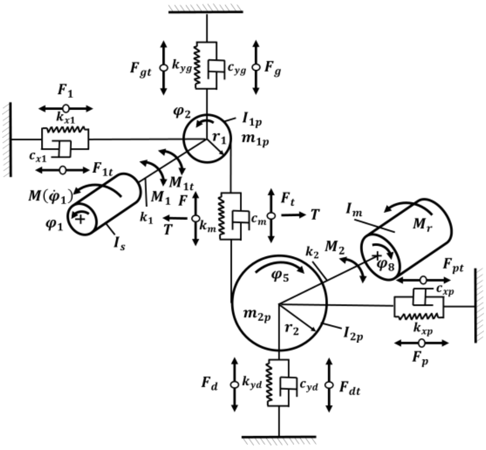
	model)	
(b)	 1-Degree of freedom model	$m_e\ddot{x} + k_e x = W$ Where, k_e is equivalent to mess stiffness of contacting teeth, m_e Equivalent mass and W excitation force.
(c)	 2-Degree of freedom model	$m_{1e}\ddot{x}_1 = k_e(x_2 - x_1)$ $m_{2e}\ddot{x}_2 = -k_e(x_2 - x_1)$ Where, k_e is equivalent to mess stiffness of contacting teeth, m_{1e} and m_{2e} equivalent masses of the gear wheels
(d)	 2-Degree of freedom model	$I_1\ddot{\theta}_1 = k_m(\theta_2 - \theta_1)$ $I_2\ddot{\theta}_2 = -k_m(\theta_2 - \theta_1)$ Where, k_m is equivalent to mess stiffness of contacting teeth, I_1 and I_2 rotational moment of inertia
(e)	 1-Degree of freedom model	$m_e\ddot{x} = k_e[x - e(t)]$ Where, k_e is equivalent to mess stiffness of contacting teeth, m_e equivalent mass and *e(t)* is the error function (it also works as an excitation function)
(f)	 2-Degree of freedom model	$[I]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{F(t)\}$ $[I] = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}$, $q = r_1\theta_1 + r_2\theta_2 + e(t)$ $K = k_m(t) = k_o + \sum_{i=1}^{n} k_k \cos(i\omega_m t +)$

Table 2 Geared rotor Dynamic model [76–78,80]

(a)	 1-Degree of freedom model	It is a two-parameter model: stiffness and damping. The inertia of rotating wheels is reduced to one mass. The motion of the mass is equivalent to the relative motion of the wheels. The motion of springs with velocity v (m/s) is equivalent to the pitch-line velocity of the wheels. The stiffness of the system is represented by the different length springs. The motion of the mass does not influence the change in velocity of the actual gearbox.
(b)	 2-Degree of freedom model	$I_{1p}\ddot{\varphi}_1 = M_1 \pm r_1(F + F_t) + M_{ft1}$ $I_{2p}\ddot{\varphi}_2 = -M_2 + r_2(F + F_t) - M_{ft2}$ Where, M_{ft1} andM_{ft2} are friction forces $M_1 = k_m(\varphi_1 - \varphi_2)$, $M_2 = k_m(\varphi_2 - \varphi_1)$ $F = k_f(r_1\varphi_2 - r_2\varphi_3)$, $F_t = C_f(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_3)$
(c)	 4- Degree of freedom model	$I_s\ddot{\varphi}_1 = M(\dot{\varphi}_1) - (M_1 + M_{1t})$ $I_{1p}\ddot{\varphi}_2 = M_1 + M_{1t} - r_1(F_1 + F_{1t}) + M_{ft1}$ $I_{2p}\ddot{\varphi}_3 = -M_2 + r_2(F_1 + F_{1t}) - M_{ft2},\ I_m\ddot{\varphi}_4 = M_2 - M_r$ Where, M_{ft1} andM_{ft2} are friction forces Forces and Moments are given by: $M_1 = k_1(\varphi_1 - \varphi_2), M_2 = k_2(\varphi_3 - \varphi_4)$ $M_{1t} = C_1(\dot{\varphi}_1 - \dot{\varphi}_2), F_{1t} = C_f(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_3)$ $F_1 = k_f(r_1\varphi_2 - r_2\varphi_3)$
(e)	 6-Degree of freedom model	$I_s\ddot{\varphi}_1 = M(\dot{\varphi}_1) - (M_1 + M_{1t})$ $I_{1p}\ddot{\varphi}_2 = M_1 + M_{1t} - r_1(F + F_t) + M_{ft1}$ $I_{2p}\ddot{\varphi}_4 = -M_2 + r_2(F + F_t) - M_{ft2}, \quad I_m\ddot{\varphi}_6 = M_2 - M_r$ $m_{1p}\ddot{y}_{1p} = F + F_t - F_g - F_{gt}$, $m_{2p}\ddot{y}_{2p} = F + F_t - F_d - F_{dt}$ Where, M_{ft1} andM_{ft2} are friction forces, y describes the vertical motion Forces and Moments are given by: $M_1 = k_1(\varphi_1 - \varphi_2)$, $M_{1t} = C_1(\dot{\varphi}_1 - \dot{\varphi}_2)$ $M_2 = k_2(\varphi_4 - \varphi_6)$, $F_t = C_f(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_4)$ $F = k_f(r_1\varphi_2 - r_2\varphi_4)$,

		$F_g = k_{yg}y_{1p}$ $F_d = k_{yd}y_{2p}$, $F_{gt} = C_{yg}\dot{y}_{1p}F_{dt} = C_{yd}\dot{y}_{2p}$
(e)	 8-Degree of freedom model	$I_s\ddot{\varphi}_1 = M(\dot{\varphi}_1) - (M_1 + M_{1t})$ $I_{1p}\ddot{\varphi}_2 = M_1 + M_{1t} - r_1(F + F_t) + M_{ft1}$ $I_{2p}\ddot{\varphi}_5 = -M_2 + r_2(F + F_t) - M_{ft2}$, $I_m\ddot{\varphi}_8 = M_2 - M_r$ $m_{1p}\ddot{y}_{1p} = F + F_t - F_g - F_{gt}$, $m_{1p}\ddot{x}_{1p} = T - F_1 - F_{1t}$ $m_{2p}\ddot{y}_{2p} = F + F_t - F_d - F_{dt}$, $m_{2p}\ddot{x}_{2p} = T - F_p - F_{pt}$ Where, M_{ft1} andM_{ft2} are friction forces, x and y describe horizontal and vertical motions Forces and Moments are given by: $M_1 = k_1(\varphi_1 - \varphi_2)$, $M_{1t} = C_1(\dot{\varphi}_1 - \dot{\varphi}_2)$ $M_2 = k_2(\varphi_5 - \varphi_8)$, $F_t = C_f(r_1\dot{\varphi}_2 - r_2\dot{\varphi}_5)$ $F = k_f(r_1\varphi_2 - r_2\varphi_4)$, $F_g = k_{yg}y_{1p}$ $F_d = k_{yd}y_{2p}$, $F_{gt} = C_{yg}\dot{y}_{1p}$ $F_{dt} = C_{yd}\dot{y}_{2p}$, $F_1 = k_{x1}x_{1p}$ $F_p = k_{xp}x_{2p}$, $F_{1t} = C_{x1}\dot{x}_{1p}$ $F_{pt} = C_{xp}\dot{x}_{2p}$

2.1.2 Contact Stiffness Estimation in Gears

Contact stiffness is the result of the stiffness of meshing teeth of the gears. Mesh stiffness is the function of the number of teeth in contact, tooth geometry, application and type of load, material properties of the gear, and geometrical modification due to profile errors/ faults in gear tooth[81,82]. The back side contact gear mesh stiffness and torsional stiffness are determined[83,84]. In recent studies, the mesh stiffness is calculated by incorporating the stiffness of the lubricant between the meshing teeth[85]. The normal and tangential components of the meshing stiffness are calculated separately and then added in two combinations: (a) In series and (b) In parallel. The hypothesis of equal shear stress on the laminar element is applied, and the effect of parameters (operating and geometrical) on lubricant film is considered like operating force, speed, and tooth geometry. Then the equivalent stiffness is the final output[86,87]:

$$K_e = K_t + K_l \quad (4)$$
$$K_t = K_m \quad (5)$$

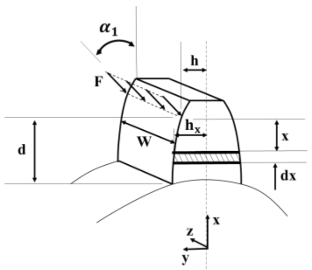
$K_l = K_n + K_{tangential}$
(6)
$K_{n/tangential} = \frac{\Delta F_{n/tangential}(t)}{\Delta x_{n/tangential}}$
(7)

Mesh stiffness of gear tooth can be estimated in the following ways[88]:

1. *Square waveform method*
In this method, for a constantly running gear-pair, the mesh stiffness is a periodic function, and a square waveform is used to approximate the mesh stiffness of the gear teeth. The period and the duration of the waveform are equal to the period of mesh period of teeth and the time duration of one revolution. The limitation of the using square waveform method is that change in contact position is ignored, there is no standard formulation for the magnitude approximation, and the reduction in the mesh stiffness is also subjective to the knowledge of the fault and its severity.

2. Potential energy method
The gear tooth is modelled as a non-uniform cantilever beam. The mesh stiffness of the gear is the algebraic addition of the bending stiffness, shear stiffness, compressive stiffness, and Hertzian stiffness. The effect of the tooth fillet was also considered for estimating the gear stiffness. The tooth fillet effect equation is helpful for healthy teeth, but the equation is not valid for faults like crack, wear, and pitting. It's pretty challenging to get the exact contact location during the rotation in gear [85]. The problem is solved by driving all the stiffnesses bending, shear, compressive, and Hertzian as a function of the rotation angle of the gear.

Table 3 Potential energy stored in meshing gear teeth[89–97]

	$\frac{1}{K_b} = \int_0^d \frac{(x.\cos(\alpha_1) - h.\sin(\alpha_1))^2}{E.I_x} dx$ $\frac{1}{K_s} = \int_0^d \frac{1.2\cos^2(\alpha_1)}{G.A_x} dx$ $\frac{1}{K_a} = \int_0^d \frac{\sin^2(\alpha_1)}{E.A_x} dx$ $K_h = \frac{\pi EW}{4(1-v^2)}$

Meshing stiffness is given by [89,98]

$$K_m = \sum_{i=1}^{n} \frac{1}{\left(\frac{1}{K_{1bi}}+\frac{1}{K_{1si}}+\frac{1}{K_{1ai}}+\frac{1}{K_{1hi}}+\frac{1}{K_{2bi}}+\frac{1}{K_{2si}}+\frac{1}{K_{2ai}}+\frac{1}{K_{2hi}}\right)} \quad (8)$$

The subscript 1 and 2 denote the driving and driven gear. The above equation can be directly used to evaluate the meshing stiffness without the root fillet effect. To evaluate the stiffness by this method requires a better understanding of the contact physics.

3. Finite element method

In finite element modelling (FEM), evaluation of gear mesh stiffness is not required. The evaluation of stiffness is required in a dynamic model. The approaches used to evaluate gear mesh stiffness are average slope (static analysis) and local slope (dynamic analysis). The models have developed liners to save computational time and cost. The mesh stiffness calculation through this method is a function of mesh density, type of finite element, and contact physics. In most finite element models, the gear rotor is modelled as a supported beam element [99].

4. Experimental method

It is pretty tricky to measure the stiffness of gear in all directions. The following factors are mandatory for measuring the mesh stiffness experimentally: stress intensity on the tooth surface and elastic deformation. The dynamic deformation of the gear surface is mapped with the help of speckle photography. The experimental methods are tested under dynamic and static conditions. The agreement between model-based and the experiment is not confirmed in any literature.

2.1.3 Damping Estimation in The Gears

The damping is a highly non-linear phenomenon and depends upon material properties, operating, and geometrical conditions[39,100]. In most gear dynamic studies, the damping is incorporated as a fixed value depending on material properties. In most cases, the value for the damping coefficient ($\xi=0.1$). The damping coefficient is proportional to the total mesh stiffness ($c = \mu K_m$) is also reported in some literature [91]. The μ is a scale constant having units of time. in a few literatures for the damping ratio estimation, the equation is further modified and expressed more explicitly as $\xi = \frac{C_m}{2\sqrt{\bar{K}_m m_e}}$, the formulation consists of the effect of equivalent inertia, average mesh stiffness and viscous damping. In gear, the damping consists of the following sources: surrounding elements, hysteresis of teeth, and fluid damping (squeeze and shear). First two are estimated with the help of the load support by the element and gear teeth, and the third one is evaluated from the Reynold's equation. The teeth damping is evaluated by hysteresis of teeth. The damping is evaluated as the structural damping or Rayleigh damping. The gear teeth are worked under the elastohydrodynamic lubrication condition in active teeth. The Reynold equation is solved for both loaded and no-load condition. The equation is solved by neglecting the axial direction losses, and if the lubricant incompressible and viscous fluid. The gear flanks are in line contact due to non-conformal contacts. So, Hertzian theory of contact plays important role in estimation of the pressure[101]. During tooth separation, the no-load condition is considered. During this period, the deformation due to the Hertzian effect is no longer applicable. Without considering the axial flow, Reynold's equation[102] is expressed as [103]:

$$\frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 12\, \mu \bar{u} \frac{\partial h}{\partial y} + 12\, \mu \frac{\partial h}{\partial t} \qquad (9)$$

In a few pieces of literature, combined stiffness and damping are derived from the lubricant film and the teeth. The dynamic pressure and film thickness are evaluated. The Reynold equation help in finding the dynamic pressure (normal and shear), the film thickness in normal compression rate, and the tangential direction speed; damping can be evaluated with the expression given as the ratio of dynamic force (normal or tangential direction) and velocity (normal or tangential direction). The damping is given as[87]:

$$C_{n/\tau} = \frac{\Delta F_{n/\tau}}{\Delta \dot{u}_{n/\tau}} \quad (10)$$

The subscript n and τ denote the normal and tangential direction. In some literature, the damping is calculated by the trace method (force-displacement curve). This method evaluates the equivalent viscous damping from the area enclosed by one complete oscillation.

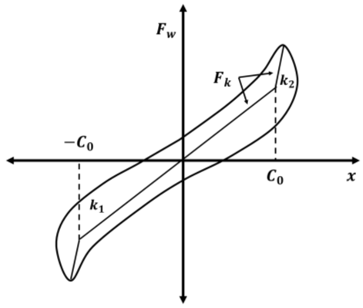


Fig. 1 Nonlinear teeth contact force and trace force function (hysteresis loop)[104]

3. Conclusions

This paper reviewed the aspects of the model to mimic the system condition based on many assumptions to simplify the modelling and reduce computational cost.

a. The gear mesh stiffness evaluation equations are only valid for the narrow approach like single tooth contact. The equation should be broadened to incorporate the effect of the multi-tooth contact.
b. The gear mesh stiffness calculated with the lubricant stiffness lacks the rheological effect of the lubricant under the single and multi-tooth contact.
c. Most of the dynamic calculations are done by considering the gear mesh damping constant, equivalent to the material damping of the gear. The effect of non-linearity of the damping needs some investigation.
d. The lubricant's effect and rheology must be incorporated in the meshing stiffness evaluation.
e. The hybrid faults on multiple teeth need some investigation.
f. The effect of the faults of the supporting element in the system, like bearing faults and shaft faults, needs some investigation.
g. Their limited models are available based on viscoelastic fracture mechanics. Most researchers assumed the gear body as rigid, leading to some errors under high-speed or high-frequency excitation.
h. The model for time-varying excitation sources deserves some level of investigation.
i. A model with an optimum degree of freedom must be defined with minimum error. The high degree of freedom leads to high computational resources, and the low degree of freedom model may lead to a significant error between the model and the actual system.

Nomenclature

C	Load stress factor

F_d	Total dynamic load on gear tooth
F_t	Tangential load on gear tooth
v	Spring motion velocity (m/s)
e	Meshing error between gear teeth
k_e	Time-dependent mess stiffness
m_e	Equivalent mass
W	Excitation force
x, y, z	Coordinate along rolling, squeezing and axial/film direction
X, Y, Z	Non-dimensional coordinate system
$e(t)$	Error function
k_m	Gear mesh stiffness
c_m	Gear mesh damping
M	Moment
F	Force
r	Radius of gear
$\mathrm{K_a}$	Axial or compressive stiffness (N/m)
$\mathrm{K_b}$	Bending stiffness (N/m)
$\mathrm{K_s}$	Shear stiffness (N/m)
$\mathrm{K_h}$	Hertzian stiffness (N/m)
$\mathrm{K_m}$	Equivalent mesh stiffness (N/m)
E	Modulus of elasticity (GPa)
v	Poisson's ratio
G	Modulus of rigidity (GPa)
I	Moment of inertia (kg. m2)

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