A New Criteria on Oscillation of Linear Delay Differential Equation

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ABSTRACT. In this article, we shall particularly deal with oscillation criteria for the linear delay differential equation. We discuss the oscillation criteria for the equation of the type.

$$x'(v) = x(v) + p(v)\,x(v-\tau) = 0\,,\ v \geq v_0 \qquad (*)$$

where the function $\tau \in \mathbb{C}([v_{0\,,}\infty], (0,\infty))$, We provide modern adequate status carry out oscillation of the solution for this kind equations.

1. INTRODUCTION AND MAIN RESULTS

In the present article, we regard the linear delay differential equation

$$x'(v) = x(v) = p(v)x(v-\tau) = 0, v \geq v_0. (1.1)$$

Where the function $\tau \in \mathbb{C}\left([v_{0\,,}\infty], (0,\infty)\right), p(v)$ is a constant function $\tau > 0$.

Let $\mu \in \mathbb{C}([-\tau, 0], R), \exists$ a unique function $x \in ([-\tau,\infty), R)$ that full-fill the requirement

$$x(v) = \mu(v)\, for\ v \in [-\tau, 0]. (1.2)$$

A specific model for population growth comes to the non-linear equation.

$$x'(v) = -cx(v-1)\big(1 + x(v)\big).$$

Here the population to $1 + x(v)$. The same non-linear equation has even arise in the study of the distribution of prime number. The stability of the trivial solution of this non-linear equation depends upon the stability of the trivial solution of its linear approximation

$$x'(v) = -c\, x(v-1)$$

Another equation which has been proposed as a model for population growth and also for gonorrhea epidemiology is

$$x'(v) = f(x(v) - g(x(v-1))$$

Theorem 1. Let

$$\frac{-1}{e} < p(v)\tau e^{-\tau} < e \ (1.3)$$

Moreover, in the actual interval $\left(1 - \frac{1}{\tau}, \infty\right)$ the characteristic equation

$$\lambda = 1 + \mathrm{p(v)e}^{-\lambda\tau} (1.4)$$

has a unique solution λ. Moreover, $\lambda < 1 + \frac{1}{\tau}$. In addition if λ is this particular solution of equation (1.4) . If x is the solution of equation (1.1) with(1.2) later

$$\lim_{v \to \infty} \left[x(v)e^{-\lambda v} \right] = \frac{1\left(\mu\ (0) + p(v)e^{-\lambda\tau}\right)}{1 + p(v)\tau e^{-\lambda\tau}} \int_{\tau}^{0} e^{\lambda s}\mu(s)ds$$

The limit being approached exponentially.

Lemma 1. [7]For an equation with several delay,

$$x'(v) = x(v) + \sum_{j=1}^{n} P_j(v)x(v - \tau)$$

Where $0 \leq \tau_j \leq n$ for $j = 1 \ldots \ldots . . n$ a minimum results retains across the hypothesis

$$\tau \sum_{j=1}^{n} |P_{j\,(v)}|\, e^{\left(-1+\frac{1}{\tau}\right)\tau_j} < 1$$

The equation (1.1) although, such condition is much precise than (1.3).In the case of infinity many distributed delay found else other.

2. PROOF OF THE THEOREM 1.

To examine the characteristics equation (1.4), we think about the function F, classified by

$$F\ (q) = q - 1 - p(v)e^{-q\tau}$$

It follows from the first inequality of (1.3) that,

$$F\left(1 - \frac{1}{\tau}\right) = \frac{-1}{\tau} - p(v)e^{-\tau} < 0$$

Again using the first inequality of (1.3) we find that for all $q \geq 1 - \frac{1}{\tau}$

$$F'(q) = 1 + p(v)\tau\, e^{-q\tau} > 1 - e^{\tau - 1}\, e^{-pr} \geq 0.$$

Since $\lim_{r \to \infty} F(q) = \infty$. If follows that there is a unique $\lambda > 1 - \frac{1}{\tau}$ such that $F(\lambda) = 0$.

Put forward the second inequality of (1.3), we discover that

$$F\left(1 + \frac{1}{\tau}\right) = \frac{1}{\tau} - p(v)e^{-\tau - 1} > 0.$$

There by as a result that

$$\lambda \in \left(1 - \frac{1}{\tau}, 1 + \frac{1}{\tau}\right).$$

This in turn enable us to estimate.

$|p(v)\tau e^{-\lambda\tau}| = |\lambda - 1|\tau < 1$(2.1)

Instantly Choose $y\ \ (v) = x(v)e^{--\lambda\tau}$ as well as determine the equivalence to (1.1) also (1.2) in favor y,

$$y^1(v) = -P(v)e^{-\lambda\tau}[(y(v) - y(v - \tau)] \quad as\ \tau > 0\,. (2.2)$$

With starting position

$$y(v) = \mu(v)e^{-\lambda\tau} \quad for - \tau \le v \le 0. (2.3)$$

Such equation at the same time are comparable to

$$y(v) = -p(v)e^{-\lambda\tau} \int_{v-\tau}^{v} y(s)ds + c \quad for \ v \ge 0.(2.4)$$

With equation (2.3) where

$$C = \mu(0) + p(v)e^{-\lambda\tau} \int_{-\tau}^{0} \mu(s)e^{-\lambda\tau} \, ds \ (2.5)$$

Since $1 + p(v)\tau e^{-\lambda\tau} > 0$ we can define

$$z(v) = y(v) - \frac{c}{1+p(v)\tau e^{-\lambda\tau}}(2.6)$$

and get another equivalent problem

$$zp(v) = -p(v)e^{-\lambda\tau} \int_{v-\tau}^{v} z(s) \, ds \quad for \quad v \ge 0. \ (2.7)$$

With

$$z(v) = \mu(p)e^{-\lambda\tau} - \frac{c}{1+p(v)\tau e^{-\lambda\tau}} \ for \quad -\tau \le v \le 0. (2.8)$$

Let $M(\tau, v)$ be maximum modulus of $|z(v)| \ upto \ (-\tau, 0)$, then we will demonstrate that

$$|z(v)| \le M(\tau, z),$$

Throughout $v \ge -\tau$, Considering any $\epsilon > 0$, presume (for contradiction) that

$$|z(v)| < M(\tau, z) + \epsilon \quad for \quad -\tau \le v \le v_1.$$

Also $|z(v_1)| = M(\tau, z) = \epsilon$ then using (2.2) we find

$$M(\tau, z) + \in = |z(v_{1)}| \le |p(v)e^{-\lambda\tau}|$$

$$\int_{v_1-\tau}^{v_1} |z(s)|ds \le |p(v)e^{-\lambda\tau}|\tau.$$

$$(M(\tau, z) + \epsilon) + M(\tau, z) + \epsilon .$$

Which is false, consequently$|zp(v) \leq M(\tau, z) + \epsilon$ for whole$v \geq -\tau$,

Since ϵ is arbitrary,

$$|z(v)| \leq M(\tau, z) .$$

across$v \geq -\tau$. Hence prove this result.

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